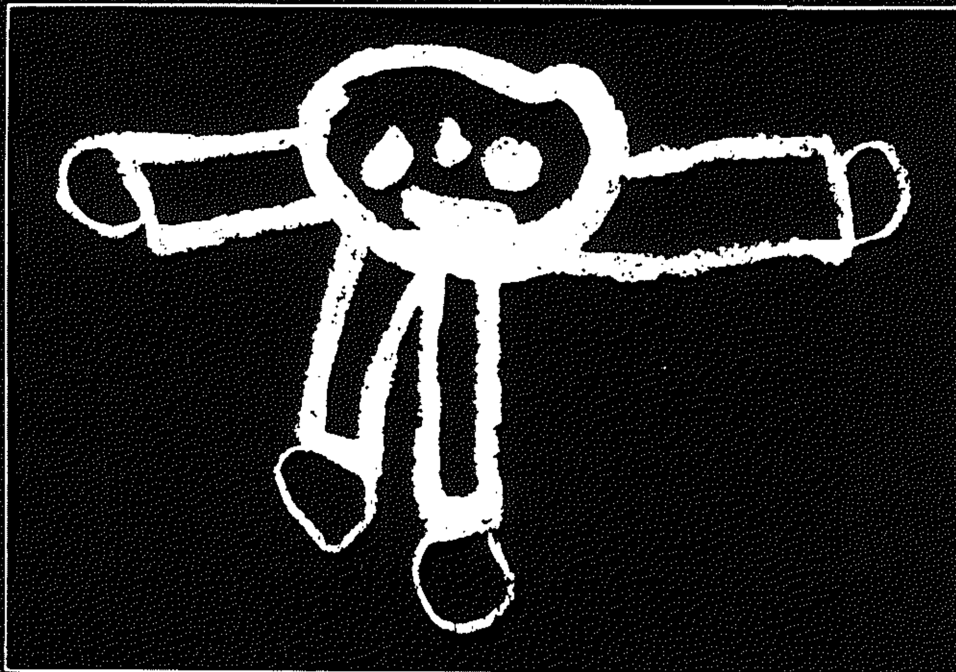
North Dakota Study Group on Evaluation



Maja Apelman

David Hawkins Philip Morrison

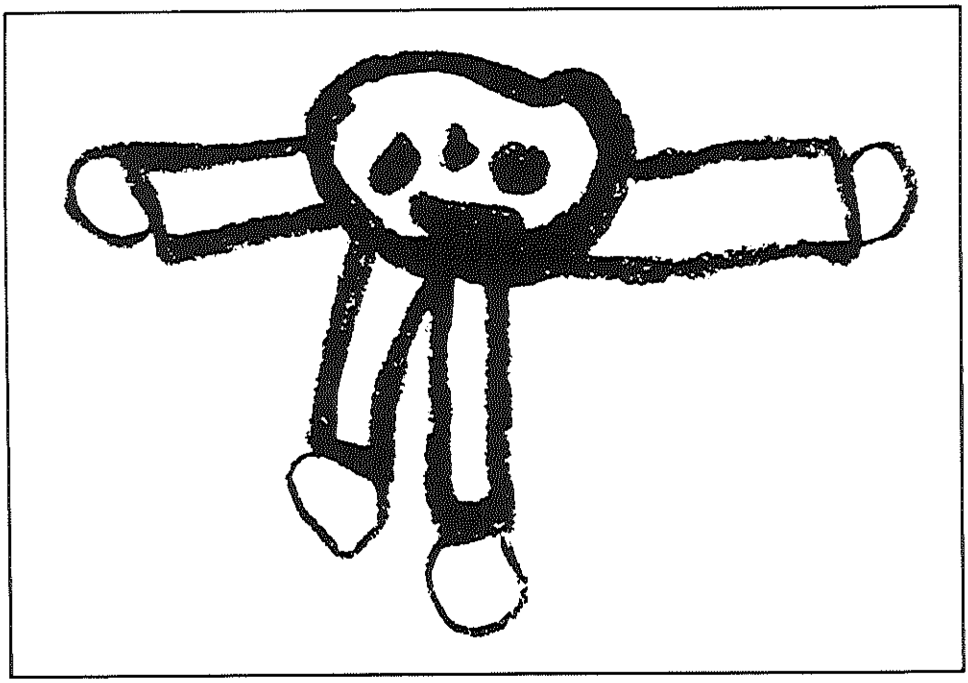
CRITICAL BARRIERS PHENOMENON IN ELEMENTARY SCIENCE

In November 1972, educators from several parts of the United States met at the University of North Dakota to discuss some common concerns about the narrow accountability ethos that had begun to dominate schools and to share what many believed to be more sensible means of both documenting and assessing children's learning. Subsequent meetings, much sharing of evaluation information, and financial and moral support from the Rockefeller Brothers Fund have all contributed to keeping together what is now called the North Dakota Study Group on Evaluation. A major goal of the Study Group, beyond support for individual participants and programs, is to provide materials for teachers, parents, school administrators and governmental decision-makers (within State Education Agencies and the U.S. Office of Education) that might encourage re-examination of a range of evaluation issues and perspectives about schools and schooling.

Towards this end, the Study Group has initiated a continuing series of monographs, of which this paper is one. Over time, the series will include material on, among other things, children's thinking, children's language, teacher support systems, inservice training, the school's relationship to the larger community. The intent is that these papers be taken not as final statements--a new ideology, but as working papers, written by people who are acting on, not just thinking about, these problems, whose implications need an active and considered response.

Vito Perrone, Dean
Center for Teaching & Learning,
University of North Dakota

North Dakota Study Group on Evaluation



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David Hawkins   Philip Morrison

CRITICAL BARRIERS PHENOMENON IN ELEMENTARY SCIENCE

Center for Teaching and Learning
University of North Dakota
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Contents

Editor's Note

All but one of the essays in this monograph -- Philip Morrison's -- are pared down versions -- some more, some less -- of material published elsewhere as reports or journal articles. As originally, they describe aspects of an inquiry that, in its wholeness, seeks to define the nature of one of our most radical educational deficiencies. What has been brought together here is a piece of research, its theoretical underpinnings, and a commentary by an external observer that suggests some directions for further inquiry. The assumption of further inquiry is a critical issue, here as elsewhere. As David Hawkins has written, "a radical may criticize and even denounce, but he must come to terms with the fact that in his self-appointed task he takes his society to have a conscience he can stir, an intelligence he can appeal to, and some wisdom he can learn from."

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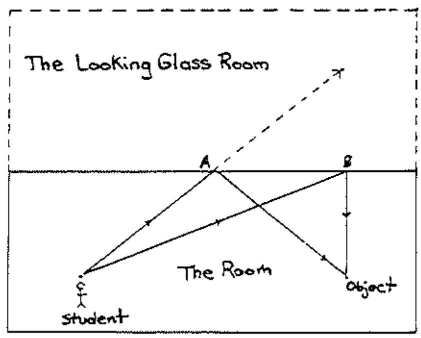
*The Nature of the Problem**

*The first part of this chapter was excerpted from a paper written by David Hawkins in 1977 for the National Science Foundation. It was done as part of a study attempting to define research into the area of scientific literacy. An edited version of the full paper appeared in OUTLOOK, No. 29, Autumn 1978. Part Two is excerpted from an essay, "Conceptual Barriers Encountered in Teaching Science to Adults: An Outline of Theory and a Summary of Some Supporting Evidence (NSF, 1982).

I. *Elementary Difficulties*

To investigate the ideas that people have about mirror vision I have sometimes asked subjects to imagine that one wall of the room we are sitting in is a large mirror and then to draw, on a map of the room, the direction in which they would look to see a given object "in the mirror." If the object requires an oblique viewing angle I have found that subjects draw a wide range of directions which cluster bimodally near two extreme positions. There is a small peak clustered near the direction which geometrical optics requires (*A* in the accompanying drawing) and a larger peak near the place defined by a line drawn perpendicularly from the object to the mirror (*B* in the drawing). For this larger



group of subjects the mirror image of the object is apparently thought of as analogous to the picture of the object pasted onto the surface of the mirror "where it would see itself," or else the depth-dimension in The Looking Glass Room is radically foreshortened in subjects' conception of it. I have found approximately the same statistics with upper elementary school children, elementary school teachers, and two graduate classes in the philosophy of science. About fifteen percent cluster at the small, correct peak, fifty percent at the other extreme, and the rest scatter in between. In the two graduate classes the fifteen percent were mostly students in physics or mathematics, while the rest were from sociology, philosophy, psychology, etc. The prevailing adult conceptualization of

From 1949 until his retirement in 1982, David Hawkins was Professor of Philosophy at the University of Colorado. For the two years 1962-64, he served as Director of the Elementary Science Study, an elementary school science curriculum project in Watertown, Massachusetts. He was Director of Mountain View Center for Environmental Education from 1970-82. His books include *The Language of Nature, an Essay in the Philosophy of Science* (1964, 1985), *The Informed Vision: Essays on Learning and Human Nature* (1974), and *The Science and Ethics of Equality* (1977).

mirror vision, and vision in general, is a gold mine of the kind of phenomena I want to discuss here.

This difficulty in understanding mirror vision illustrates a class of what I shall call critical barrier phenomena, or simply critical barriers, in present-day science teaching. These phenomena are easily observed in many contexts and represent barriers to learning for at least a clear majority of precollege, college, and adult students. Though diverse in content, the phenomena share certain characteristics which I believe are uniform enough to sustain some reliable generalizations.

They appear early in any standard science curriculum; they are associated with extremely "elementary" science topics. I put the word in quotes because "elementary" is often taken to mean "easy" or "obvious," and thus appropriate to begin with. In fact, as I shall try to show, some "elementary" ideas are exceedingly *un*obvious to those who have not yet assimilated them and are themselves only lately-won in the history of science. Elementary ideas are often deep. Students who fail to assimilate them must often come to regard them as barriers to entry into any further science learning. Often they give up, becoming frustrated and typically either dropping out or dropping up -- that is, continuing the course and managing to pass it without any valued or valuable precipitate of understanding.

Some students manage to avoid this impasse. They may have had early, self-directed interests and talents; they may have had early successful teaching. They have already assimilated and can readily use elementary ideas which, for others, are formidably opaque. In the few cases where I have some recorded statistics, this group is small and typically consists of those who already have a conscious bent toward science as a career or an avocation.

My concern is with the general level of science education, not with the advanced education of scientific specialists -- it is with the size of the base of a social pyramid, not the height of its peak, though I am mindful of the relation between the two measures. I believe that by carefully examining the class of critical barrier phenomena it is possible to arrive at some conclusions about present levels of scientific culture and modes of science teaching at all except the highest levels. These conclusions do not automatically define remedies, though they suggest some. My concern is, rather, to use them to define goals for science education policies, goals which I believe are crisp and definite enough to suggest useful criteria for decisions about ways of working toward them.

In the following section I shall further illustrate, define, and interpret the class of critical barrier phenomena. In the final section I shall attempt to define policy goals I have in mind. Chief among these is the need for much more basic research, analysis, and experimentation. The experiments with mirrors, and others I shall cite, were casually done and should be repeated with more carefully stratified sampling of subjects. I believe I am describing only the tip of an iceberg.

Further Examples of Critical Barrier Phenomena

My other examples of critical barriers come from contexts as rich and illuminating as the mirror difficulties. They are size and scale, air and water, heat, and elementary mechanics.

Size and Scale

In some fifteen years of teaching a general physical science course for non-science college majors, and in an equal period of time devoted to inservice teaching of general science to elementary school teachers, I have found in both groups a marked conceptual difficulty in grasping, or gaining fluency with, the elementary relations between length, area, and volume. The frequency with which this difficulty appears (if one looks for it) is high; it affects something in the neighborhood of eighty or ninety percent of both groups. Reasonably patient explanation is no cure. For this reason, a teacher concerned to "cover the subject" -- meaning, of course, to get through a textbook or promised outline -- will become exasperated with students' disabilities or with his own inability to make such elementary things clear. The fact that patient explanation is no immediate cure is a hallmark of the class of critical barrier phenomena. One *can* break through but not easily or uniformly, and failure may lead a teacher to say that some people are just dumb. Another hallmark of the class, however, is that when the breakthrough does come with students they often have a high emotional release, a true joy in discovery; "Is *that* what it means?" There is often a marked change in later performance, as though a hitherto hidden secret had been revealed.

Returning to length, area, and volume (L, L^2, and L^3) and their relations to each other, both the resistance to explanation and the subsequent joy of discovery suggest that the student is not lacking in knowledge so much as he is habituated or addicted to some congenial alternative way of thinking. My work with children of middle- and upper-elementary ages reveals that with materials, time, and supportive interest they can arrive at these relations through honest empiricism -- not yet firmly built-in, perhaps, but without confusion or conflict. Generalization may be difficult; it is one thing to see the scale-relations with larger cubes built out of smaller cubes and quite another to recognize them in the scaling up or down of spheres of different sizes, of irregular shapes of dough or plasticine, of models, of doll houses. That takes time.

Let me inspect this example in terms of what we call common sense or common knowledge. Length, considered in isolation, is no problem. But, though farming, carpet-laying, and painting involve area, area is indeed problematic, especially in relation to length or as a characteristic of irregular shapes. "You can't find the area of a footprint; it's not a rectangle." An eight-by-ten rug is an eight-by-ten rug but "is it really *eighty* square feet?" Volume (*pace* Piaget) is well understood in typical adult volumetric contexts but *not* as L^3.

With such shaky foundations the next steps -- the principle of similarity and the elementary scaling relations -- are quite inaccessible. And here I think I have support from history. The ancient Greeks had formulated these ideas. Euclid establishes them formally, though we would say awkwardly. Galileo was the first to elucidate their relevance to the properties of material things in his discussion of the strength of beams. Extended to include time and mass they are implicit in Newton and were more or less formulated in nineteenth century physics. D'Arcy Thompson in the early twentieth century was, so far as I know, the first to see living things as phenomena of scale. At about the same time Lord Rayleigh elaborated dimensional analysis as a style of simplified physical analysis.

I mention this history because it shows how long a time was required for a simple and widely illuminating idea to show its full implications, even among the learned. At a relatively elementary level the now-old P.S.S.C. (Physical Science Study Committee) physics text for high schools was written in the spirit of the scaling laws. Philip Morrison, one of its co-authors, gave the Christmas lectures at the Royal Institution of London on this subject for British school children some years ago. Ironically, the P.S.S.C.'s opening chapter on the scale of nature has been eliminated from commercial editions, apparently because these ultimately simple considerations which give rough intelligibility to the whole face of nature are still not considered to be physics.

I have used this example of a critical barrier phenomenon of science as part of my introduction to the discussion of our failure to achieve a wide dissemination of scientific ideas and attitudes; it suggests that we are up against something rather deep in the relation between science and common sense; we are up against a barrier to teaching in the didactic mode which has hardly been recognized, or if recognized has been seen mainly as a challenge to ingenuity in teaching rather than as a challenge to a deeper understanding of human learning. It is the sort of phenomenon we tend to acknowledge only in a spirit of despairing humor or complaint; we tend not to focus on such matters as worthy of intellectual curiosity and excitement. Why *are* these difficulties at once so elementary and so abundant? That question is too seldom asked.

As a first step of analysis, I suggest that the verb "to learn" implies a time scale; some things can be learned in five minutes while some come only on a long developmental time scale. It is the great merit of Jean Piaget to have emphasized the importance of the latter kind of learning, and his great demerit to have popularized the belief that what takes place is not really learning at all but an age-specific biological development independent of a society's educative potential; if an intellectual skill or scheme cannot be taught, just wait a while and it will appear anyway. This does not reflect Piaget's best thinking but he has never repudiated it. The whole class of barrier phenomena I am concerned with here represents the apparent inability of most adults in our society to get beyond what would have to be classified as limitations that belong to early stages in the Piagetian taxonomy.

With respect to length, area, and volume, most adults have something in mind that is quite different from, and potentially conflicting with, the geometrical sense for invariance and variation in scale. They have a perceptual-commonsense way of taking things as "big" and "little" without reliance on the analytically defined concepts of length, area, and volume. From the commonsense-perceptual point of view this is entirely reasonable. Immediate commonsense judgment is geared to a great variety of perceptual cues and its practical reliability is typically very high. Since over the range of normal experience length, area, and volume are highly correlated, it is plausible that in the commonsense scale of big and little there is for most practical judgments of size no focal consciousness of any one of them. When challenged to measure the area of a footprint, most students, most adults, will suggest measuring *around* it, measuring its perimeter. The notions of perimeter and area are not clearly distinguished from one another.

In order to compel attention to such distinctions there are many ways of using the principle of the extreme case, such as artificial or naturally occuring shapes with large perimeters and small areas, large areas and small volumes, etc. This is not only an exercise; it leads naturally to the many biological examples of adaptation to scale -- the roots and leaf area of green plants, the elaborately branching lung tissues and guts of large animals, etc. In extending curiosity and experience to these ranges of phenomena -- many of them everyday phenomena accepted incuriously by common sense -- ordinary incurious perceptual habits of thought can be gradually cross-linked to those which are more analytic and more consonant with the newly extended range of experience for which the history of science is responsible.

A deepening grasp of the significance for scale of invariance and variation is one of the major gateways to the modern world of science. It represents the acceptance of an intellectual discipline upon the extraordinary subtlety and pattern-recognizing capacities of ordinary perceptual learning, capacities that are geared to the great variety and complexity of the human world and are basic to many forms of understanding and of art. In such perceptual matters the axiomatic simplicity of geometrical scale is by itself almost useless; yet in extending our knowledge and intuition of the ampler world of science -- with which our life as a society must be increasingly concerned -- the failure to develop these axiomatic thought-habits and to link them fluently to perceptual modes will inevitably rob the mind of a power it increasingly needs. The failure to grasp the planetary impact of present-day activities and practices -- the failure to understand what it means to scale an explosion by a thousand or a million -- can be fatal to a society. Beyond that, however, it is a failure which robs most of us of the possibility of any esthetic and moral framework within which we can understand and enjoy, and thus be full participants in, the great and problematic era our history has created. Without it most of us will remain or increasingly become what Arnold Toynbee called a "cultural proletariat," in but not of the society we unwittingly constitute.

Air and Water and Beyond

My third critical barrier phenomenon of present-day science education has an equally interesting and varied history. It is the scientific conceptualization of the states of matter. Aristotle sorts them out as a matter of course, with fire suggestive of our "energy." Nothing is more obvious, and common sense has no immediate trouble with the traditional introduction to the elementary text which began with the sorting into solids, liquids, and gasses. Yet here again a large majority of "non-scientific" college students and adults develop deep difficulties. Let me begin with the atmosphere. We live in it like the fishes in water and its very constancy as the medium of our life renders it mainly unnoticeable except for special circumstances which common sense recognizes in its usual piecemeal perceptual fashion. From history, again, we know that scientifically "obvious" things about the air are recent in any human consciousness. The Greek astronomers appear to have deduced the "ocean of air," a terrestrial mantle of limited thickness. This, I believe, was to explain the remarkable fact that an object so distant as the moon (whose diameter and distance they had fixed from the geometry of the eclipse data) was still clearly visible, while distant mountains, so close by comparison, were almost lost in the atmospheric haze. At any rate, Plato weaves a myth around the ocean of air. Yet the impact of the idea -- otherwise long forgotten -- came back to scholars full force only after Torricelli's and Pascal's investigations and the visible fact of the Torricelli vacuum.

The elementary school science text or demonstration can prove that air has weight, and usually does so badly, with balloons, avoiding the consideration of the ocean in which the weighing is done, of buoyancy and density. High schools can evacuate a flask weighed before and after, and that is a neater demonstration; but neither demonstration can produce any resonance in a mind which is unprepared, as most are. The siphon is a familiar phenomenon on the edge of everyday experience, but for most of the group I speak of it is another of those mysteries which is only deepened by patient scientific explanation. Elevate the top of a water siphon to thirty-odd feet, a silly trick just beyond the edge of common experience; now the sense of mystery at the result will become palpable.

We often discuss, pro and con, the educational impact of television. News programs are characteristically climaxed by a discussion of the national and local weather, complete with those marvelous satellite pictures, accounts of new "systems" moving in or out, of the jet stream, of highs and lows. Some, at least, of those weather experts are indeed good meteorologists, but like many scientific experts they have long since forgotten what most of their audience does not know it needs to learn, the early slow steps by which they themselves assimilated a conceptual structure which meteorology already presupposes. I discussed this once with a TV weatherman, a good meteorologist indeed, and suggested some televised byplay with water barometers, rotating dishpan models of the atmosphere, and the like. He thought it would be fun but explained that time constraints required rapid speech and bare daily essentials. Yet today good climatologists are raising questions about man's own impact on the climate. What sense will these concerns make to intelligent citizens for whom the global circulation of air and water is unreal -- for whom water evaporates and condenses only up and down, locally, and for whom, half the time, air is literally nothing, half the time reaches on to the moon, and all the time is mysteriously able to support the flight of airplanes?

Another aspect of this topic concerns the elements of biochemistry and their relation to the green cover of the globe. For thousands of years farmers have farmed well in the belief that their crops are earth-earthy, pushing up from the maternal soil and somehow composed of it. Water and the heat of the sun were necessary but the stuff of life came from below. That view, like some Jungian ancestral memory, still dominates the thought processes of most of us. It is only a few generations since there was a scientific realization that trees are essentially shaped from air and water, that sunlight drives their circulatory systems, that they grow from the outside in. A large majority of our adult students will tend to believe the opposite: that plants -- grasses or trees -- push up out of the ground, their blades or branches slowly rising, their newest growth in the center, and all this despite a forgotten course in biology.

At a slightly more sophisticated level are the ways of conceptualizing the interphase characteristics of things, the simplest and most accessible being the water-air or water-oil boundaries. The idea of a "skin on water," being of negligible significance on the human scale, is hardly credible to common sense, though intelligent discussion of it often raises up the phrase "surface tension" from some otherwise forgotten science lesson. This leads nowhere. Soap films are not credited with thickness and their colors are rarely provocative. Here again scale is of the essence and a sense for it is lacking. Evaporation and condensation -- up and down -- are believed in separately but are not understood as shifts of equilibrium in an always two-way exchange.

The missing ingredient here is any insistent realization of atomicity. Atoms are known about in the verbal store as something to be believed in but not as things to be imagined in conceptualizing everyday physical, biological, or chemical processes. The simplest reasoning of John Dalton, or even of Lucretius, is again a critical impasse for most; explanation only heightens the impasse, though such now-accepted terms as "carbon monoxide" and "carbon dioxide" are familiar.

Here again there is ample historical evidence of the recency of such ideas and of the discrepancy or unresolved conflict between the scientific and the commonsense-perceptual modes of thought and imagery. The everyday physics of qualitative change is still predominantly in the mode of the early Aristotelians and alchemists, a metaphysics of dispositions and qualities -- thus drying, cooking, dyeing, melting, dissolving.

Heat

Heat is another critical area, with temperature as associate. Thermometers are historically recent but are widely assimilated into the commonsense world. For most, what they measure is perceived as a refinement upon Aristotle and the medical investigators of Galileo's time. Temperature is a measure of "temperament" in human bodies and outside, of the balance between two principles called the Hot and the Cold. This ancient conceptual predilection is indeed a nice match to the animal temperature sense, which measures something which is *not* physical temperature, although correlated with it. What it measures is approximated by the scientific notion of heat-flow, in or out, but at this level common sense conflicts with any notion of heat as substance, whether in the early form of "caloric" or the modern one of thermal energy.

This congenial notion of the Hot and the Cold conflicts with physics so long as we fail to recognize that here again the commonsense-perceptual categories are inherently a different sectioning of experience than that of modern science, more discriminating for many of the purposes of common life but less significant as abstractly universal. This commonsense notion of the Hot and the Cold can be mapped into the scientific framework only after we know a great deal not only about physical heat and thermodynamics but also about the temperature sense, its linking role in the homeostatic regulating mechanisms of the animal body and its purely psychological aspects. If the physical concept of heat appears to common sense as inaccessibly recondite, the commonsense notion of heat can be represented scientifically only by a complex and perhaps still incomplete model. The transformation from one conceptual domain to the other is not one-to-one, is not simple; it is one-to-many and many-to-one. Here as elsewhere, of course, the scientific concept has its roots in common experience and thought, but the steps by which it has evolved took two centuries or more of analysis and research, reaching into the last decades of the nineteenth century.

Elementary Mechanics

Historically the earliest modern science, mechanics is beset by many similar pedagogical troubles. Even the idea of balance of forces, which goes back to the Greeks and is treated as a dull little subject introductory to the older texts, is in fact a fascinating thicket of these troubles. Archimedes derived the law of the balance from pure considerations of symmetry, by a style of argument which is powerful and deep, anticipating that of Liebnitz and of modern theoretical physics; it is close to common sense but not as a formal intuition, not for predicting the stability or instability of structures made of wooden blocks or Tinkertoy. Almost none of our subjects knew ways of thinking about the stability or instability of balance. In this context the image of the center of mass lying at the bottom of a potential well or on top of a potential hill is radically difficult to reach. This does not imply the technical vocabulary I use; the image can be that of a marble in a bowl, but the linkage of analogy is unavailable. Similarly, the transition from Aristotle to Galileo in the discussion of motion is equally unavailable. For perceptual common sense, motion is always and inevitably in a medium; air may not be thought of as real but space is definitely full, not empty. Mechanics derives Stokes' law for falling bodies by adding a resisting medium to Galileo's law. Common sense, like

Aristotle, has to go the opposite route, but it abhors the distinction between air and the vacuum.

Mechanics is full of examples of things which for most of my subjects are unteachable by standard means and which, if so taught, hardly go below the level of verbal discourse and artificial problem-solving. They certainly do not become what Piaget called *schemes*, penetrating to what Dewey calls "the subsoil of the mind." Common sense says that the wall does *not* push back on me when I lean on it; the flight of the airplane moves nothing downward to keep the plane up. Perhaps the textbook science is stored for a while in some basket of recall but much of this learning can be unlearned; it is not irreversible.

An alert and apparently very lively college sophomore had what appeared to be incurable difficulties with the idea of the relativity of motion. The context was that of an introduction to astronomy, but homely examples were to no avail, nor was patient explanation after class. An imaginative tutor finally got the student to pirouette counterclockwise while observing the walls and ceiling, inviting her to imagine that she was stationary and the room rotating clockwise. After two or three trials it suddenly worked, with the characteristic high emotional release. In this case the change was major and unusually dramatic; she moved from failing grades to a very adequate final paper on the kinematic equivalence relation between the Ptolemaic and the Copernican models of the solar system. It is not always so simple; students more often must relive such transitions repeatedly. A teacher for whom kinematic relativity is second nature may fail entirely to grasp the intellectual nature of this difficulty or to understand that explanation with diagrams, no matter how patient, inevitably presupposes the very conceptual transition it seeks to explain. Historically, we are reminded that even Galileo did not describe inertial motion with full generality of context, but only on a horizontal plane. The thought experiment which requires a body moving arbitrarily in empty space was apparently not available to him.

Levels in Learning

It seems evident that in considering these critical barriers we must avoid a confusion of *levels* in learning. In many cases less obvious than those discussed above verbal structures are often received and in some ways assimilated by students. These structures may be returned on examinations or even applied to the solution of simple problems but what has been so learned does not prove retrievable or applicable in new situations, especially those arising outside of class or in later years. The loss rate of isolated knowledge transmitted in science classes is often about equal to the rate at which the knowledge is gained. The partial recognition of these problems is very old, probably as old as formal instruction, but somehow they have not been brought into sharp focus.

It is not appropriate to discourse here about the psychology of educationally significant learning, for which in fact we have no widely received and powerful theory. It is, however, appropriate to distinguish between learning conceived of as the reception, retention, and recall of verbally coded and transmitted information, and learning understood as the development of intellectual habits for *transforming* sensory or verbal information to bring it into congruence or conflict with prior general knowledge or belief. The critical barrier phenomena suggest that it is this latter kind of learning which has failed to take place. If such matters have been taught in a superficial way -- verbally transmitted, momentarily understood, and retrievable as fact but not transformed into tools or disciplines for further learning -- then loss or burial is unavoidable. A teacher who had been taught about the conservation of mass, in high school or college, could maintain without conflict that a terrarium sealed for seven years now weighed more than when she had planted and sealed it "because the plants are bigger." She could be reminded of her earlier learning, but only very slowly did she acknowledge, with final delight, the logical quandary involved.

I have deliberately emphasized the prevalence of learning failures of the most elementary kind, but such failures also occur at higher levels, even among the scientifically learned. The very high energies of cosmic rays were for a long time regarded as a prime mystery. Only two or three decades ago Fermi pointed out that dynamic equilibrium between stars and free atoms in space would imply even larger cosmic ray energies than those observed -- the principle of equipartition. Suddenly, as a result of Fermi's observation, the question was reversed; why aren't the cosmic ray energies larger? In a recent popular television program on man-powered flight, many fine technical details were mentioned, but no one thought to dramatize the simple fact that even a bird geometrically scaled up to the mass of a human being couldn't fly. The difference between a bird and the man-powered Gossamer Condor is of a piece with the anatomical contrast between mice and elephants. These two examples are at very different levels of scientific knowledge and sophistication but the latter was as unavailable to the learned of three centuries ago as was the former to those of three decades past.

I have emphasized elementary examples for several reasons. First, they are commonly overlooked prerequisites for even the kind of basic scientific culture we deem necessary to life in our present world. Second, what is elementary from a scientific point of view was often unavailable even to the learned of a relatively recent past. "Elementary" should not be thought of as meaning easy or innately understandable. A sense for powerful elementary ideas is not the beginning of scientific knowledge but is typically a late product of its evolution. Individual learning does not have to recapitulate history, but history can tell us a lot, commonly overlooked, about the dimensions of the learning and teaching tasks we face.

A third reason for my emphasis is to combat the commonly received notion that widespread scientific education and culture is increasingly problematic because of the vast increase in scientific knowledge, which allegedly requires specialization beyond any layman's possible understanding. But the power of even simple scientific ideas, fully mastered and enjoyed, can make the scientific world-picture intelligible overall and in first approximation, and that is the level at which I believe we have mostly failed. How else can we understand the prevailing level of PR about something called the neutron bomb?

Since the immediate purpose of this essay is to propose a definition of goals sharp enough to suggest directions of search and research into means of achieving these goals, I think it is proper to emphasize still further the distinction between the two levels of learning mentioned above -- the "verbal structure" level and the level of true conceptual understanding, of easy insight. It has been the historical aim of science both to extend our experience and to reduce it to order, these two aspects being always interconnected. The ancient astronomers -- early Greek or pre-Greek -- extended their experience by carefully mapping the sky and its motions over centuries. At some point this suggested, or allowed, the strange notion that Earth was not an indefinitely extended cosmological boundary but a thing, a body, perhaps a sphere, poised in space. This was proposed as a fact which fitted all the data, but it was much more; it was a reduction to order of many otherwise unrelated astronomical phenomena.

But this new order conflicted with commonsense intuition, which required a universal cosmological up and down. The conflation of these two ways of thinking created the uneasy question about why *the* Earth, now a body among bodies rather than a cosmographic division, didn't fall, and a question about upside-down inhabitants of the antipodes. Even Dante put the entrance to Hell *down* there. A century or two after the early Greek discoveries, Aristotle announced, with a lingering note of triumphant understanding, that "down *is* toward the center." The round Earth-body was not simply a new fact to be stored along with other facts; it was a fact which required a radical reorganization of the whole category structure of geographical and cosmological thinking. If it were taught merely as a fact, without appreciation of the need to help it penetrate into the subsoil of understanding and to rebuild the mind's category structures in the process, it would remain something merely bookish and abstract, to be entertained nervously and then forgotten.

II. Sagacity, Learning, and Taxonomy

Recent research on learning, problem solving, and cognitive development has brought us back, with renewed vigor and -- perhaps -- fresh insights, to some of the oldest problems of the theory of knowledge. For the purposes of our present research it will suffice to go back to the end of the nineteenth century, to William James' classic *Psychology*.* James' discussions of thinking and of reasoning are all pertinent to the present-day concerns of science and mathematics teaching, and more generally to the investigation of the cognitive procedures of children and adults.

*James, William, *Psychology*, 2 vols., Henry Holt & Co., New York, 1899. The most relevant chapters are VIII, IX, and XXII.

James makes an initial distinction, reflected in many languages by contrasting verbs or nouns (*kennen* and *wissen*, *connaitre* and *savoir*, etc.), and uses a Jamesian terminology to mark this contrast: *acquaintance with* versus *knowledge about*. Acquaintance-with implies familiarity, recognition. As a mode of knowledge, it is not attributive, not propositional in character. We always in principle *know* something *about* the things we are acquainted with, but this implies a focusing of attention and effort of analysis which mere acquaintance does not require. We are acquainted with some persons and not others, as with some situations and places. James' emphasis on and uses of this distinction prepare the way, in his subsequent chapters, for a certain dichotomous tension which appears in different guises in different contexts: between perception and conception, between particulars and universals, between the peripheral and the central, between the concrete and the abstract, the intuitive and the analytical, the figural and the formal. When James turns, in vol. II, to the treatment of reasoning (our current jargon would usually, more narrowly, say problem-solving), he places an essential emphasis on the contrast -- related to that of acquaintance-with versus knowledge-about -- between *sagacity* and *learning*.

James' discussions of reasoning is cast partly in everyday psychological language and partly in terms of traditional Aristotelian logic. Reasoning is about something, it has a subject, a subject presented for thought. The process of thought is one of predication or attribution. The resources for thinking are always thus dual in character. The first resources are presentational (typically perceptual), the second are those somehow retrieved from the thinker's fund or store of knowledge. In making this obvious first move, James is aware of a question which is often overlooked. What is presented as subject for thought is typically a concrete thing or situation, a logical *particular*. But it is, and is perceived as, a particular of some *kind*, as exhibiting a logical *universal*. In order to retrieve or recall anything about the subject for thought, we must recognize in it, and single out, some universal character or trait, or implicitly grasp some similarity to things previously encountered which we can recall. This ability James, following John Locke, calls *sagacity*. If we think in terms of the mind's filing system, the perceived character of the subject is what directs us to some relevant file or files. In computer jargon it contains the *address* of such a store.

The outcome of thinking is learning: what is retrieved from the store and reliably fitted to the new situation is itself presumably what had once been learned and stored there. What is thus freshly learned is then also added to the store. Thinking is an interplay between *sagacity* and *learning*.

James' use of the term *sagacity* suggests, as it is intended to, that any concrete particular subject of thinking, attended to *sagaciously*, is attended to *as* an instance of some essential universal category: it is not only seen, but *seen as* an instance. This latter notion, of seeing-as, was recently emphasized by Wittgenstein,* who wished to avoid the paradoxes which can result when the perceptual and conceptual aspects of thought are too sharply separated. However veracious and undistorted our perception may be, it is always a partial and selective affair, thus made ready to fit some interpretation which context and habit make likely. And however abstract and formally defined our conceptual apparatus may be, it is always linked to some diversity of perceptual or figural material, to the imagery of recalled or imaginary perceptual experience, to *intuition*.

*Ludwig Wittgenstein, Philosophical Investigations, New York, 1953.

When the linkage between the perceptual and the conceptual (or the particular and the universal) is very strong and immediate, we approximate the kind of experience in which reasoning plays no part at all. Most of our daily performances, if observed and catalogued minute by minute, would appear to be of this kind, routine, habitual, unthinking. The stimulus and response are one seamless fabric. What presents itself for perception is *immediately* recognized and responded to in some more or less appropriate way, as a whole of meaning, familiar and unproblematic. "A rose is a rose is a rose..." There is no reasoning involved.

It seems reasonable to recognize, within experience, a sort of continuum, ranging from such virtual automatism at one extreme (in which things are taken at what we call face value and responded to unthinkingly) to a predominance, at the other extreme, of uncertainty, of awareness of novelty, of recognition of the problematic, and thus on occasion of the supervention of a new level of activity, of more or less systematic and analytical thought.

James' recognition of the sagacity-learning linkage invites us to a kind of taxonomic, and thoroughly Aristotelian, view of what Plato called coming-to-know, of the development of our ability to learn, to store what is learned, to retrieve what has previously been stored, to return what has been freshly added to the store and, as a not infrequent consequence, to work at reorganizing the storehouse itself.* In our present view, all of the important kinds of learning difficulties we (as reflective teachers and researchers) have encountered can be described within this scheme.

*cf. Hawkins, D., "Taxonomy and Information," *Boston Studies in the Philosophy of Science*, III, 1966.

James' emphasis on the notion of sagacity, therefore, raises a question which is absolutely central to all of our concerns about teaching and learning. How is it, and under what circumstances is it optimally possible that we are able to gain access to previously stored information which will, in new and problematic situations, help us on our way to problem solutions or, more generally, to fresh understanding? And how is it that, in the wake of failures, we are sometimes able, with help, to reorganize some parts of the store, to add to them fresh experience, and so to find successes?

In the Aristotelian scheme, a mind's fund of knowledge is organized *per genus et differentiam*, as a taxonomy, as a filing system in which each genus is subdivided by differentiating characteristics into two (or more) sub-genera. The defining characteristics of each taxon, each genus, are chosen, but only more or less adequately, to be those which are *essential*; that is, to be just those characteristics which are most reliably associated with many others. Thus in Aristotle, the category *man* is a subdivision under *animal* distinguished by *rational*. The genus *biped* and the differentia *featherless* would equally well distinguish us from the other animals, but would not provide or sustain a rich or logically coherent taxonomy. Very little important information about humans would be related merely to our bipedal status, and even less to our lack of feathers. Animality, on the other hand (we moderns would qualify and say mammal, or even primate, animality), supplemented by some label which recognizes our special capacities for communication and learning, would provide a far more useful (because more coherent) organization of what we know about man's place in nature. Such central attributions are "of the essence" in Aristotle's scheme.

James' distinction of sagacity and learning is related to a classic discussion of Aristotle (e.g., *Metaphysics*, Book I, Ch. 2). In discussing the nature of wisdom, he defines a sort of continuum of things knowable, lying between two extremes. At one extreme are those things which are "most knowable by us" and at the other are those which are "most knowable in themselves." The latter are exemplified by first principles, laws, universal truths. They are first in the order of importance, but last in the order of learning. Because of their abstractness and universality, they relate to and are involved in defining the most generic features of that which we experience. What is most knowable by us, on the other hand, coming first in our experience, is the world of the concrete particular phenomena and the wide diversity of quite specific categories into which these fall.

Common Sense to Science - Category Shifts

One of James' central arguments concerning the categorization of experience is a kind of modern relativism which Aristotle would not share or find relevant to his concerns. James recognizes the possibility of many different taxonomies, insisting that what we regard as the essential characteristics of things is wholly relative to the dominant purposes for which we use or take account of them. By way of comment on Jamesian relativism versus Aristotelian metaphysics, it is useful (and sufficient for our present study) to take note of a characteristic contrast, in implied purposes, between the commonsense organization of knowledge, its category structure, and that of the sciences.

Commonsense categories tend to be defined (implicitly) by relatively accessible characteristics and, where these are not reliable indicators of other consequential traits, we tend to define by *clusters* of such characteristics, figurally rather than formally;* by some kinds of perceived pattern-similarity, "family resemblance." To quote Eve, in Mark Twain's story of the newly created Garden, "It just *looks* like a camel." We are told that Massai herdsmen can recognize and sort each others' cattle, in large herds, without resort to branding. Apparently personal ownership becomes reliably associated with pattern-differences across a large diversity of subtle variations. The reliable, rapid reading of a printed text for understanding is perhaps an achievement of the same order.

*See Jean Bamberger and Donald Schön, "The Figural ↔ Formal Transaction," DSRE Working Paper WP-1, MIT Room 20C-124, June 1978.

Another characteristic of commonsense categories is the generally loose logical organization in which they are related to each other. Thus, larger classes are usually recognized and identified only by practically accessible characteristics or clusters. So we can often recognize and identify some individual species of plants, but their genera and families are totally unrecognized unless they happen to be grouped and distinguished by some simple or obvious traits. But these in turn are often at variance with the scientific groupings, analogous rather than homologous. Thus a whale is a fish with a horizontal tail, as it says in *Moby Dick*. Such groupings are obviously relative to characteristic human interests and purposes. So also, of course, is the alternative classification of whales as cetacean mammals rather than as any kind of fish. It is difficult, however, to regard the existence of such alternative ways of classifying as a demonstration of relativism when so little of closely observed whale behavior is fish-like, and so much else fits the description of a mammal evolved to be ocean-going. In Aristotle's language, this second description "divides nature at the joints," which the first to a degree does not. Thus we can admit that classification is relative to the purposes for which it is organized, but we can observe, all the same, that some classifications can serve a far wider variety of purposes than others including, most importantly, the further pursuit of knowledge.

The history of science may thus be regarded as a history in which the filing systems of commonsense knowledge are (a) deliberately expanded in content, (b) reorganized (sometimes radically) to accommodate this expanding content with minimal redundancy, and in which (c) each of these commitments is deliberately chosen as a guide to the pursuit of the other.

The whale is one example of this kind of category shift. Another example of this process, characteristic but very simple, arises in the elementary understanding and terminology of plant anatomy. Having previously recognized the distinction between simple and compound leaves in such obvious cases as the locust, and having observed the universality of the bud at the axil of the true leaf, one is then obliged to say that what common sense would immediately recognize in shape and size as leaves (e.g., in the Kentucky Coffee Plant) are really only leaflets, small parts of the true leaves, which common sense would in turn call branches. In this reconstruction one sees an absolutely characteristic scientific motive, which common sense does not often share or need to share -- a motive of loyalty to *universals* (in this case of plant anatomy and development), many of which are verifiable only by far closer examination than our normal prescientific interests would sustain.

A more complex case is one which surrounds the concept of *metal*,* as that has been reconstructed throughout the history of chemistry and physics. The common sense, ancient conception of metal can be more or less identified with some cluster of relatively obvious characteristics (shiny color, heat conductivity, etc.), a definition by family resemblance. The twentieth century scientific conception, on the other hand, is rooted in that of an atomic crystal lattice structure in which the outer electrons are very easily detachable within the lattice, forming a kind of "electron gas." This conception is defined in terms of characteristics which are radically inaccessible to common macroscopic experience, but which, when defined, allow an extensive and precise elucidating of properties, linking these into the generalities of quantum physics.

*cf. John Dewey, *Logic, The Theory of Inquiry*, New York, 1939, Ch. IV, passim.

If we now return to the Jamesian style of discussion we may construct a very skeletal account of levels or phases of thought (reasoning, inquiry) involved in the use, retrieval, and reorganization which is implied by this sort of transition from common sense to scientific categorization. To carry out this discussion, we extend a scheme which James only mentions in passing, that of the traditional patterns of the syllogism. The extension lies in the use we make of these patterns, as follows:

1. In the case of automatic or unmediated recognition we write: $\underline{S} \longrightarrow \underline{P}$ (Sagacity), meaning simply that some presented situation S is *seen as* an instance of P and responded to directly, with no further thought.

2. In the next level this operation of sagacity occurs, S is *seen as* of some kind, M. But M is now not of itself sufficient to close inquiry; it serves instead as an *address* to a whole file of things labeled M; and from that file is drawn a generalization: Things called M *may be relied upon to have the property P*. With this discovery the inquiry is closed:

$$\begin{array}{l} S \longrightarrow M \text{ (sagacity)} \\ M \longrightarrow P \text{ (learning)} \\ \hline S \longrightarrow P \text{ (closure)} \end{array}$$

This is, of course, the traditional first figure of the Aristotelian syllogism. In the logic text, this figure (called subsumption) looks rather trivial: "Socrates is a man, all men are mortal, therefore Socrates is mortal." If, however, we are simply using this pattern to *describe* a common pattern of reasoning, it fits well enough. The new situation S is *seen as* a case of M, but the M in question may not be quite the same -- family resemblance -- as those past cases of M which have supported the generalization that all M's are P. In this case we might write

$$\begin{array}{l} S \longrightarrow M' \\ M \longrightarrow P \\ \hline S \longrightarrow P? \end{array}$$

and thus indicate that an element of judgment and hazard (as to the importance of the difference between M' and M) may be involved. Whether this judgment is finally confirmed or shown wrong, the content of the file M will have been altered by the inclusion of a new instance, and also therefore by the strengthening, rejection or redefinition of some generalization based upon the collection of instances of M.

In fact it is clear that the pattern

$$\begin{array}{l} S \longrightarrow M' \\ M \longrightarrow P \\ \hline S \longrightarrow P? \end{array}$$

leads very simply to a consideration of *analogy*. M' being no longer taken immediately as identical with M, we examine to see if the identifiable *differences* between the present M' and past instances of M are or are not relevant -- whether the *grounds of analogy* are weak or strong.

In the case of biological taxonomies, these alternatives are often represented by a contrast between *analogy* and *homology*, between accidental similarity and similarity due to common origin. So the whale and fish have some analogous characteristics, but the pentadactyl limb is one telltale homologous link between whale and land-mammal. Biological influences based on mere analogy may be limited and superficial; those based on homology are deep-going. The Tasmanian wolf is not a wolf but a marsupial, analogous to the wolf in appearance and habit, but only far more remotely homologous. When common sense habitually and dogmatically classifies by superficial appearance, we may call this habit the Tasmanian Wolf syndrome.

3. At a third level, having seen that S is M'; and having discovered that the file M is *not* a useful guide to thought, or to further inquiry, a next possible step is more careful examination of S itself, in which S is seen finally, to be of a kind genuinely different from M -- say, N. N is now the index to quite another file in the store of knowledge, and the process starts over again.

A simple example is the transition, mentioned earlier and discussed more fully below in the investigation of the hot-cold contrariety, from this kind of polarity to a conception of heat as a physical substance *of some kind*; under this newly-tried category, one can now conjecture that there will be a *quantity* of heat in any material thing which could approach zero and thus, also, *imply* an absolute zero of temperature.

4. The most characteristic use of analogy in thinking arises when there appears to be *no* general file category related to our most sagacious perceptions of the situation S. S now first presents itself as uniquely novel. Lacking any general category to fit it in, we can look for other *particulars* we can recall from memory; these may in turn create new direction for file-searching. Thus a mathematical problem, not soluble by any known algorithm, may remind one (on casting about) of *another* problem, superficially dissimilar, which *has* yielded to some particular method of solution. The first problem is now tentatively *seen as* an instance of some quite different trait, no longer as an S that is M or M' but as a T which is thereafter found to be an instance of R; so the file-search process is begun all over again.

As a result of this kind of reconstruction, the acquisition of scientific knowledge creates a special difficulty -- that scientific concepts often form an interconnected network. Any *one* such concept is to be understood as a node in a network involving *other* scientific concepts, tightly interconnected. Commonsense concepts, by contrast, are often loosely connected, since they are defined by some readily observable traits, or by family resemblances among clusters of such traits. Such concepts are thus relatively less dependent on their logical interrelations to each other, more readily established one by one, "most knowable by us."

In examples such as the solid-state physics theory of metals, one can observe the very great organizing power of theory (electromagnetism, the quantum atom, crystallography, thermodynamics, for example), providing as it does a relatively small collection of conceptual tools which guide the description of nature over very wide ranges. At the same time, however, one can find at least a partial explanation for many of those apparently rudimentary learning difficulties with which our research is concerned. Scientific concepts (in contrast with those of common sense), however illuminating when well-understood ("most knowable in themselves"), often form a network which is strongly interconnected. *To understand any one concept, a node in the network logically connected to other nodes, it is necessary to understand many others as well.*

This logical tightness in the network of scientific ideas, their mutual interdependence, suggests immediately a paradox; they *cannot* be learned; not in isolation from each other, not all at once, hence not at all. Such a paradoxical conclusion only states, in extreme form, the origin of many of the student difficulties.

Model Building and Model Testing for Critical Barriers

As the foregoing discussion implies, our work has tried to describe, and in a sense explain, some characteristic difficulties in the learning and teaching of science, and to do so in a language which discusses phases and transitions of experience, building models which represent what we have called critical barrier phenomena. Such models purport to describe *interior* processes which will be reflected in learner's observable behavior.

One question about such thought-models is whether they can, as hypotheses, be adequately tested by empirical data, or whether *they will remain only ad hoc*, speculative accounts.* We give no dogmatic answers to this question. Our efforts at confirmation of some models has been informal, exploratory. But we think we have turned up some interesting *clusters* of phenomena, some of which we have been able to predict from others on the basis of fairly simple explanatory models.

A second question about such thought-models concerns their usefulness as guides toward the improvement of teaching. In the twentieth century history of psychology, such accounts of thinking and learning as we here offer have often been disparaged as "introspective" rather than behavioral. As this applies to our work, we reject the label. It would be better to say that the models we develop are models which impute to our students the same capacities which are implied in our own investigative performance. When, for example, we try to learn about learning we -- as researchers -- exhibit the strengths and limitations of *our* own sagacity and our own learning; the conceptual apparatus and language we make use of is of the same genre as that of the persons we study. Their thinking may be more naive than ours in some ways, but it does not differ in kind.

*cf. Robert Davis, "The Postulation of...Frames." *The Journal of Children's Mathematical Behavior*, Vol. 3, No. 1, Autumn 1981, pp. 167-201, esp. pp. 167-170.

We impute ways of thinking to them which we ourselves can try to practice and report on, and which can give guidance to our teaching.

If we find significant results in this mode then also, we believe, these results are directly useful for other teachers who have understood them. If many students have difficulties in the same ways and around the same subject matter, thoughtful teachers will attempt just the same kind of model-building which such research as ours can pursue further, in a more careful way. This will be to the further benefit of teachers' diagnostic and planning abilities.

The general outline given above -- in the name of Jamesian psychology -- is not logically tight enough or detailed enough to be called a testable theory. It is a necessary sort of plausible framework of *conditioning* assumptions. It directs us to look for the typical source of critical barriers in the reconstruction of mental filing systems, those which scientific understanding requires; and in the kinds of category-shifts which this leads to, which can then be consolidated as a basis for greater scientific understanding. In the following material we have set forth examples, both from previous teaching experience and from current (and more carefully documented) research. In some cases we merely describe certain characteristic learning-difficulties. In a few others we go further in developing and giving evidence to support some of the kinds of unifying hypotheses which can help explain a range or cluster of such difficulties.

Teaching: The Art Essential to the Research

One final methodological comment is suggested by the above account of thinking. Our conditioning assumptions imply that the most persistent and basic difficulties in the acquisition of scientific knowledge are difficulties which involve not so much a sheer *lack* of information as a trouble in making category shifts, shifts which involve a *reconstruction* of ways in which experience is codified and filed away, and then later retrieved. If this is a correct assumption, then an essential phase of the teacher's art must be conceived of supporting and seeking to guide the learner's own reconstructive commitments and efforts. Where such shifts are needed, a teaching style which simply transmits *more* scientific information, *more* knowledge, into the learner's already established but inappropriate taxonomic scheme, may prove radically inefficient, even pedagogenic. If the learner's taxonomy is seriously at variance with that which the teacher's instructional efforts presuppose, only verbal and conceptual conflation and confusion can result; only troubles of a pedagogenic order will accrue.

In just this way, I do not believe the ideas I have tried to organize in this essay would have come to definition without occasional success in the science-teaching art itself; an art in which I have learned much from others more deeply involved in that art than myself. The art is essential to the research.

Once a barrier phenomenon has been observed in the course of teaching it is often possible to investigate it further by means external to the classroom. Such investigation can in turn contribute to the refinement of teachers' own diagnostic skills. But an investigator who is also a teacher has access to other sorts of information hard to come by outside a tutorial or classroom environment. This is the evidence of success, the evidence that a diagnosis has been correctly matched by the nurturant provisions a teacher has made on the basis of diagnosis. Failure is evidence also, and in teaching it counts equally as it leads to revised diagnosis and fresh provision.

For the above reason it seems urgent that the research I recommend should not be too often or too long detached from the teaching ambience. If my analysis of barrier phenomena is at all correct significant teaching success in helping students overcome such barriers has a time-scale intrinsic to it which is not that of the ordinary laboratory or the journal article, though it can fruitfully involve them. This means that skilled and insightful teaching is indispensible to the research, that the normal academic boundaries between teaching and research must be broken down.

Clearly such research must be highly interdisciplinary, combining resources from philosophy, from developmental and cognitive psychology, from those deeply versed in subject matter and in its history, and from practitioners who teach -- the last-named being especially crucial to the investigation needed.

I would suggest, finally, that in the course of such research the history of science education itself, in philosophy and sometimes in substantive detail, could fruitfully come under review. I would anticipate that studying earlier curriculum development efforts could contribute to this research but the research should not be aimed prematurely at providing better teaching strategies, better curricula, better supporting materials. It should be research aimed at producing greater understanding, stimulating wide discussion and a wide multiplicity of practical trials and developments. Like many other teachers, I have views about what some of these trials and developments would be and have been a propagandist for them. Here I have put such views aside in order to emphasize that we need a rather radical reconstruction of our own ideas about the nature of science and of effective science teaching. The common sense of teaching experience is where we must start; we simply have not extended it far enough, nor do we yet know how to reduce it to order. I offer the critical barrier phenomena as a major clue and a promising start.

2

*The Research Project: Learning About Size and Scale**

"Critical Barriers to the Learning and Understanding of Elementary Science" was the title of a research project directed by David Hawkins at the Mountain View Center, University of Colorado, from 1980 to 1982. The project grew out of a talk Hawkins had given at MIT in 1976** in which he had mentioned the need for

a radical reconstruction of the organization of scientific knowledge, a reconstruction designed to make science maximally penetrable from outside, to make it more readily accessible either by minds whose powers are first developing or by minds which have developed patterns other than those now deemed apt for science.

He added that he wanted to search for and define "those almost irretrievably elementary stumbling blocks which pedagogy normally sweeps under the rug because it does not understand them . . ."

In a policy statement written for the National Science Foundation in 1978,*** Hawkins described in detail a group of problems encountered in his own science teaching, and named these problems "critical barrier phenomena." He noted that some so-called "elementary" ideas are "exceedingly *un*obvious to those who have not yet assimilated them:"

We are up against a barrier to teaching in the didactic mode which has hardly been recognized, or if recognized has been seen mainly as a challenge to ingenuity in teaching rather than as a challenge to a deeper understanding of human learning.

In his formal proposal to NSF, Hawkins defined critical barriers in more detail:

1. They are conceptual obstacles which confine and inhibit scientific understanding.
2. They are "critical" and so differ from other conceptual difficulties in that they:
   a. involve preconceptions which the learner retrieves from past experiences that are incompatible with scientific understanding.

*This chapter, by Maja Apelman, appeared in slightly different form in OUTLOOK, No. 45, Autumn 1982. It was written originally for Mountain View Center's report to the National Science Foundation on the NSF-funded research project on "Critical Barriers to the Understanding of Science."

**Talk presented to a symposium in honor of Victor F. Weisskopf, published in *Physics and Our World: A Symposium in Honor of Victor F. Weisskopf*, American Institute of Physics, New York, 1976. Reprinted in OUTLOOK, No. 22, Winter 1976. The quotes are from the OUTLOOK article.

***The opening section of this monograph is an edited version of that policy statement.

Maja Apelman started her educational career as a preschool teacher in New York City. Later she taught Head Start paraprofessionals at New York University as well as early childhood curriculum courses at Bank Street College. In 1971, she moved to Boulder, Colorado, to join the staff of the Mountain View Center. There she worked primarily as a classroom advisor until she participated in the Critical Barriers Research. She is now active as a freelance educator -- teaching, writing, and consulting.

b. are widespread among adults as well as children, among the academically able but scientifically naive as well as those less well educated.
c. involve not simply difficulty in acquiring scientific facts but in assimilating conceptual frames for ordering and retrieving important facts.
d. are not narrow in their application but when once surmounted provide keys to the comprehension of a wide range of phenomena. To surmount a critical barrier is not merely to overcome one obstacle but to open up stimulating new pathways to scientific understanding.

Most of the data for the Critical Barriers Research came from two semesters' work with small groups of elementary school teachers who came to the Mountain View Center weekly for two-hour work-and-discussion sessions. We were looking for teachers who had little or no formal science background but who were interested in learning science both for their own sake and for the sake of the children they were teaching. We also wanted teachers who were interested in becoming more aware of their own learning styles and willing to talk and to write about them. We worked with 10 teachers each semester, three of them attending both semesters' seminars. A few teachers we did not know but most of the group had previously worked with the Mountain View Center.

Courses at Mountain View, when it was operating as a teachers' resource center, were informal: small classes, generally no more than 10 teachers, lots of work with materials, and plenty of time for questions and discussions. The research courses were similar in format. We* generally started with a discussion of the previous class, worked with materials in small groups, and came together at the end of the session to share impressions, raise questions, and plan for the following week. We asked teachers to write brief notes at the end of each class -- "immediate reactions" -- hoping to get further clues for planning the next session. We also asked for longer, more reflective notes to be handed in at the following class, and for a final evaluation at the end of the course. These notes, together with the transcripts of the classes (and, during the first semester, transcripts of the staff planning sessions) constitute the major part of the raw data of the research.

*The research staff in the first semester consisted of David Hawkins, Ronald Colton, and Maja Apelman. In the second semester, Abraham Flexer joined the group.

My role in the barriers research was twofold: I was to be participant observer in the classes, learning with the teacher/students, but at the same time keeping alert to any non-voiced difficulties they might be having and encouraging them, partly through my own question-asking, to speak up in class. I also attended the staff planning meetings and gave my colleagues feedback from my informal talks with the teachers as well as from interviews I conducted. In addition, I had the opportunity to get help with my own learning problems between classes. I found that if I had just gained a new understanding I could assist teachers still struggling with similar problems. Having only just surmounted the stumbling blocks, I was closer to the teachers' problems than were the other researchers.

Although I loved my work, there were times when I found it difficult to be of value primarily for my ignorance. I believe, however, that such an intermediary role of student advocate is an important one in this kind of research or in any class in which adult students are struggling with difficult "elementary" concepts.

The Substance of Scientific Explanation

David had decided to begin the first semester's course with a study of Size and Scale,* partly because of his own interest in this subject and partly because of his previous experiences with students' difficulties in this area. "Size and scale is a pseudostructure for the seminar," he said to me. "If you have length, area, and volume in place, you have a useful ladder to understanding things from atoms to galaxies."

*The second topic of the first semester was Heat, on which we spent five class sessions. Eight sessions were devoted to Size and Scale.

Soap Bubbles

At the first meeting of the class, David started right in with the main topic of the course. He asked why people have trouble with what surface area means:

This is a very common source of intellectual difficulty, and not just because of the mathematics. There seems to be a situation where you don't have to do much computation and still you get into trouble. Why would that be? What is there about ordinary, everyday human experience that would make it difficult to disentangle? Area, surface, it's around us all the time. It is the sort of thing that intrigues me, because it doesn't seem plausible that it should be that difficult.... If people don't understand something, it's not because they are stupid, it's because they have preconceptions which they have learned, and often learned under conditions which make these preconceptions usable and useful. Then we require that people give them up. It's like giving up a friend.

He explained further that his interest was in students' difficulties, in finding out how a person was thinking, in the so-called naive questions people might ask. "Imagine being valued for that!" Hedy, one of the teachers, commented. In a few short minutes the tone of the class was established. Hedy probably voiced the feelings of most of the teachers present, and David was convincing in communicating to the teachers that he was deeply interested in their individual ways of thinking, of learning, and ultimately of understanding. "This is the complete reverse of what is ordinarily valued, and it's one of the main reasons why these fascinating naive questions have never been seriously looked at," he said.

After further discussion about teaching, learning, and research, Mary wanted to know how we would go about getting past critical barriers once they had been identified. "That implies that we look at specific topics and get into the substance of a scientific explanation," David answered. "Let me give you an example." He got an egg beater and a bowl of soapy water, prepared earlier for use during the class.

We started beating the soapy water, watching bubbles form, multiply, diminish in size. The whole mixture grew in volume while air was being beaten into it, but when we continued beating, it got stiffer. David drew our attention to the change in the mix:

There is an awful lot more internal surface of little bubbles, there are more and more little bubbles and if you could measure the total amount of surface where there is liquid and air in contact with each other, it increases and increases and increases. There is a big change in the ratio of volume of water to the surface area that the water has in common with air...and the bigger you make the surface per unit volume, the stiffer the stuff gets.

I now wonder whether any of us knew then what David was talking about. Hedy wanted to know if there was more "surface air," and I said that I was so mixed up I didn't know any more "what's water, what's air, and what's soap." David explained that the beaten-up solution was a mix of a liquid and a gas that acted as a solid, adding that we had to start thinking about what it was that made this difference. "It is a problem related to just the amount of liquid film that is in there."

Still puzzled, I asked for a parallel example that might help me understand what happened to the soapy water. Ron mentioned water drops, comparing big drops with the tiny drops found in clouds. "The big drop of water doesn't have any stability." What is a stable water drop? I wondered: "Does stability mean not breaking apart, or not being real strong?" Instead of an answer, I got a promise that we would play with water drops on wax paper in the following class so we could study and observe how different-sized drops looked and behaved. I don't know why the concept of stability gave me so much trouble, but it came up again and again during the early classes and staff discussions.

Soap bubbles were one real-world example with which David and Ron wanted to introduce the topic of the relationship of surface area to volume. Ron had also brought in a dead plant with a root, pulled up from his garden. He pointed to all the little hairs at the end of the root, stressing the enormous amount of surface in contact with the soil and the water. We talked about what happens when you transplant a plant and cut off the tip of the root which has most of the fine root hairs. Sally found this example helpful, "but those bubbles," she said, "I just have to take your word for it, I don't picture it at all." I didn't understand the bubbles either, but the surface area of the root hairs presented me with another problem. "I don't ever think of roots as having a surface," I said. "I think of their length but not about their surface area. I don't look at these roots and think: if I put them all together they will make so much surface area." "But every tiny bit of water has to go from some place in the soil across the surface of the root to get inside the plant," David said. This is so obvious when you know and understand it.

In fact, the term *surface area* gave me problems. When I asked if other teachers had trouble thinking of an area as being made up of "long strings of things" (intestines had been mentioned, the elephant's huge gut), Mary said she wasn't sure she had trouble with that but she wondered why she should care about it. "Why is it important?"

I finally asked if we could just talk about *surface* and leave out the word *area*, which seemed to cause my confusion. David suggested that we talk about "the amount of surface." I liked his acceptance of my problem and his willingness to change the terminology. This is another way of validating a student's thinking.

Since this was our first meeting, most of the teachers weren't ready to raise questions as freely as I was. But asking questions and voicing my confusions was what I was expected to do. My questions were always genuine. The teachers sensed that and gradually became freer in exposing their own troubles.

We did not ask for homework that first week, but two teachers, Hedy and Sally, brought back some notes. Sally wrote:

...How do they know that the mixture becomes more solid because *the surface area of the bubbles becomes greater, thus creating more strength? And how do they know that increased surface area in relationship to the volume creates strength?.... I'm taking someone's word for it as I always have in science. Could you actually see the surface of these bubbles through a microscope and analyze the relationship between surface area and volume? Where is the starting point for this understanding?*

"How do they know..." expresses a feeling common to non-scientists. Scientists' knowledge is far beyond our understanding. We have never known how to gain entry into their world because such entry has been offered only on the scientists' terms, which have been incomprehensible. So we remain outside, wondering "How do they know?"

Hedy had made a list -- "off the top of my head" -- of all the things that seemed problematic to her. It was a kind of confessional, alerting the staff to some of her troubles:

--*Estimating volume: I'm aware of this when I try to decide which size container to put left-overs in. I invariably choose containers that are too large.*

--*Diagrams* -- yech -- *especially verbal descriptions of spatial relations. I have a physical desire to crawl beneath a table.*

--*Percentages are almost incomprehensible to me. That dates from 8th grade.*

--*Geometric formulas -- utterly meaningless. Square roots, light years, same.*

--*Light spectrum and photography I have never grasped. Many concepts of physics and astronomy are interesting to me. I enjoy hearing the explanations but I never retain any of it. It's like a fairy tale only I can't remember how it goes.*

Water Drops

The agenda for the next class included: playing with drops of water, using wax paper, eye droppers, and food coloring set out for that purpose; more experimenting with beating soap froth as well as egg whites and whipping cream; observing how oil behaves when dropped into water; and floating needles and razor blades. I didn't know what all of that had to do with size and scale but that didn't bother me.

David's favorite way of preparing for a class is to experiment with the materials he wants to use, and so I got a chance to play around in advance with the razor blades and needles, learning that the needle will float if it doesn't get wet, which sounded strange to me, and that if you rub it against your skin it will get oily and float more easily. "The insects which walk on water keep their feet dry," David said. Do insects have oil on their feet? I wondered. I now know that there is a delicate balance between the pressure of the water spider's foot and the surface tension of the water on which the little insect puts its weight. But at the time of the course, the idea of the water's elastic *skin* was not yet part of my mental framework.

We had decided to start the class with the planned activities. Everyone began by playing with the water drops on wax paper. Sandy wrote in her notes: "My first reaction was surprise that such simple materials could be so interesting. The investigation was so open-ended and nondirected." There are many careful observations of water drops in the teachers' notes. Ann wrote, "It's fun to watch the small bubbles be 'gulped up' by larger bubbles. The drops seem to pull together. The small bubbles seem more round and almost have a peak compared to the larger bubbles." Hedy, whose initial reaction to our set-up was negative -- "I won't like it" -- nevertheless allowed herself to get involved: "I am amazed by how self-contained each bit of water is. I am delighted by the movement of the drops. As I pull them around they change shape -- amoebalike. They have charm." I, too, found the water drops fascinating as they slithered over the wax paper without leaving any sort of impression. "How come the water isn't wetting the surface it is on?" I wondered. "Well, that must be the nonmixing of wax (oil) and water. So they really do not mix." Personal observation is more convincing than a teacher's explanation.

At one point David filled an eyedropper with soapy water and announced that he had a "mystery liquid": just the slightest trace of that liquid on the surface of a water drop made the drop disappear. That raised a question in Hedy's mind: "If soap makes water strong enough to hold a bubble shape, why does it make water drops on the paper lose their shape?" That apparent paradox bothered me, too. How could soap destroy a water *drop* but strengthen a water *bubble*?

After a while, some teachers moved on to another part of the room to beat the various liquids we had prepared and to experiment with the oil and with floating needles and razor blades. Others remained with their water drops, continuing their observations. Beating soap froth prompted Mary to write: "I wonder about mixing air with liquids. Since air is invisible, I have difficulty imagining what must be actually happening." Mary found it hard to think of *invisible* air. I thought of air as a kind of *nothingness* that fills the spaces between real objects. During one conversation, David had described air as having molecules with space in between. I found that amazing. How can there be space in between, when I think of air as *nothing* to begin with? My *nothing* view of air was a real stumbling block to my understanding of soap froth. The millions of little interfaces which increase the internal surface area, and thus strengthen the mix, troubled me for quite a while. No wonder! I thought of *real air* which interacted with the soap film as *surrounding* the entire soap froth, but the *inside* air had apparently remained in my *nothing* category.

The teachers who had remained with the water drops became fascinated by two discoveries: the magnification of the print on newspapers with which we had covered the tables, and an inverted reflection in the tiny water drops of a large skylight. Both Sally and Sandy wrote about the skylight, and both added interesting comments about their feelings as students of science. Sally wrote:

I was most fascinated with the way the drops magnified anything under them and the way a bubble in a drop demagnified anything under it. We discussed magnification with a convex lens and demagnification with a concave lens. That seemed clear enough. Is it really that simple?

The rest of the time we looked at different objects through various lenses. Print became inverted as we pulled the magnifier away from it. But I was puzzled about the skylight: it was always inverted. Why? I went home wanting an answer (we had had so many questions and so few answers) and so I looked in the encyclopedia. I think I found a partial answer: "An object being examined through a magnifying glass is always kept at a distance from the lens that is less than the focal point. If the object is at a distance greater than the focal point of the lens, an inverted image is found." I assume that our lens was always at a greater distance from the ceiling than its focal point, thus the image was always inverted. So what is the focal point?

It's exciting to think I might have observed something and learned from the observations. But after all that looking and questioning, why is it that I wonder if I've drawn any correct conclusions? When I see a math pattern emerging, I'm excited and confident about continuing it. I know I'm on the right track, but in science I don't have the background to know if what I think makes sense.

Sandy also wrote about her discovery of the inverted skylight in the water drop. But it wasn't the inversion that intrigued her. She wondered

*how such a large area as the ceiling could be continued on such a small surface as a water drop. I was shocked and perplexed at this phenomenon and dismayed by the fact that no one else was equally astonished.... How quickly I abandoned that track of thought when it did not meet with shared interest; or, when I sensed that it might be the least bit "obvious."**

*It's too bad Sandy did not pursue her interest. But in a class where people are encouraged to go in their own directions, one student's need for feedback may come at a time when others are too involved with their own observations to respond. If Sandy could have brought her question to David's or Ron's attention, she probably would have had a different reaction.

Hedy wrote of her feelings when she asked David a question about the change in the water drop's surface tension when soap was added. I don't know what David said to her but this is what Hedy felt: "That was not what he wanted me to be thinking about.... I just had some of that older feeling of I'm not doing it right, whatever it is.... I've got to go where he's going and I don't know where that is."

These were intelligent women and successful, experienced teachers, yet in the unfamiliar domain of science, old insecurities, fears, and frustrations quickly arose. One couldn't imagine a more relaxed, more supportive setting than the one we tried to create for these meetings, but old feelings are strong and it didn't take much to bring them out.

I want to quote from one more student's paper. Shelley gives a wonderful description of her gradually growing interest and involvement in the water drops and her ultimate fatigue after a period of hard thinking.

I started out with very random intentions -- joining bubbles, pulling, blowing, etc. -- feeling like I didn't really know what I was supposed to be doing. I quickly became fascinated by the effortless gliding of the bubbles over the paper -- it was a wonderfully quiet and relaxing activity. I thought -- this would be great after an intense, thought-provoking activity. BUT before long I began noticing patterns *-- the large bubbles pulled in the small ones; the smaller the bubble the more spherical; I could blow air into the bubble making a bubble within a bubble (what is a bubble?); the bubbles seemed to magnify slightly, but a small bubble inside acted just the opposite. These patterns began to give direction to my randomness and it didn't matter what I was supposed to be doing. I began sharing my discoveries with those around me and I became fascinated with the magnifying and "demagnifying" powers of the bubbles -- like looking thru both ends of binoculars. The little air bubbles within the larger bubble had pushed the water aside making a concave surface -- opposite of the water's convex surface.*

My quite, relaxed mood was gone and I wanted to find out more about lenses. But I didn't know where to go from here. Viewing print thru the double lens made things backwards and upside down. I don't need the answers to all this because I know I'd never remember them at this point but I seem to need some directions so that I can go further or be sure my observations are correct.

All of these things kept me so absorbed that I didn't want to join the others doing something else. I couldn't handle any more ideas!.... I left feeling stimulated but a bit drained and frustrated too. I feel like I'm at a standstill and there is so much to learn.

Shelley expresses the frustration experienced by many adults who start to study science after avoiding it most of their lives. The feeling of "there is so much to learn" can be overwhelming, especially when you work with a teacher like David Hawkins, who believes in starting with broad complexities rather than with premeasured little units which can be monitored but which rarely take you to the larger, more exciting understanding of science.

Since there had been no time for a general discussion at the end of the last class, we had decided to start the next class with a group meeting so teachers could ask questions or share experiences. David opened the discussion by expressing his hope that the teachers hadn't been thinking, "I wonder why we are doing this," but had been able to get involved with the materials. The teachers *were* involved, some more so than others, of course, but I'm sure they were also wondering about the purpose of the activities. Harriet, for example, ended a detailed account of her work with drops and bubbles with this question: "What to do now with this information?"

David knew why he wanted us to play with drops: it was another illustration of the surface-area to volume relationship. A small drop has a much larger surface area in relation to its volume than does a large drop, so the surface tension of the drop -- the tendency of the *skin* around the drop to pull together, to shrink into the shape of a sphere -- is stronger than the gravitational force which pulls the water down. A large water drop, with less surface area in relation to its volume, is more affected by gravity and therefore tends to flatten out.

This very important relationship of surface area to volume, on which the whole topic of size and scale rests, was totally lost to the class. David intended to use this relationship as a starting point for numerous excursions into the world of nature, where it plays such a central role, but we hardly got off the ground. Interesting, open-ended activities had been planned for the first class to introduce the concept of the area/volume relationship. The teachers enjoyed the activities but they also got interested in many nonplanned topics. Marsha, for example, said that she had tried to whip the vegetable oil -- with no success. Why would cream whip but not oil? Here is David's answer:

...There are lots of phenomena that you can observe and get interested in...what do you do with kids when they ask why *questions, like the child asking where living things really come from. You say to yourself, "Oh Lord, the words that I would use would not be understood," and you try to direct their attention back to more things they can learn at their level. You probably evade their questions. I can tell you right now I haven't the vaguest idea what sort of answer to give to that. I think I could begin to evade it somewhat.* Could you whip butter into foamy stuff? Butter is the fat, cream is a mixture of other things. I can see a difference between cream and oil.*

Hedy said she had a question about bubbles, the same question she had asked in her notes: if soap makes better bubbles, if it strengthens the bubble shape, then why would it make water drops collapse? This was another *why* questions which David did not answer directly. He repeated Hedy's observation, adding more descriptive details, and then pointed out an apparent paradox:

*David's *evasions* sometimes annoy me but I understand why he is doing it. He wants to direct learners' attention back to the phenomena under investigation, back to the "things they can learn at their own level." He encourages further observation and experimentation and gives just enough guidance to make progress possible. He doesn't like to supply answers which he thinks won't be understood, preferring the more circuitous, more time-consuming route that learners have to take in order to make their own discoveries.

The soap makes the surface weaker, the drop doesn't hold together as well. On the other hand, you can't make bubbles with ordinary water that will last, whereas if you put a little soap in the water, they will last. If you keep the humidity high so the bubble can't evaporate, the bubble can last for hours. Soap bubbles are stable although their surface is practically weaker than it is on water.

Instead of explaining how this can be, David mentioned similar examples of apparent contradictions: heavy cream which is thick, yet light (that had come up earlier in the discussion); and children's use of the words *big* and *little* or *fast* and *slow* to describe a variety of different attributes. This led to an interesting discussion about language, and how labeling different properties with the same word can get you into difficulties in understanding those properties. I don't know whether Hedy felt that her question was answered, but she seemed perfectly satisfied.

Mary now had a question. She wanted to know if the surface tension around the water was a substance separate from the inside of the drop. David acknowledged that the outer surface looks like a different substance but explained that it isn't:

It just acts differently. When it's down in the interior it acts one way and when it's out where half of its neighbors are not there, it acts in another way.... It's the same substance but at a boundary it behaves differently from the way it behaves inside.

Mary surprised quite a few of us with her next question: "Isn't it that way because of the hydrogen bonding?" David agreed that it has to do with the way water molecules interact with each other but he added, "Let's stay out of the molecular world for just a while because we want to stay on a more elementary, a child's level." Rather than enlarging on the role of the molecules, he talked about the problem of teaching young children about atoms and molecules. David emphasized that verbal explanations aren't meaningful to elementary school children. Even with us he preferred to stay away from molecules and concentrate more on observation. That suited Hedy fine. "I really don't care why drops stay that way or any of the *whys* about it. The main thing is that it's pretty or it's pleasing and it's really an effort to care why." I remember Jean and Sandy nodding in agreement.

I cared very much about the *whys* and I was still puzzling about the paradox of soap weakening water drops but strengthening bubbles. David had called soap bubbles more *stable* and had said that using the word *strong* got us into trouble. I didn't understand that. Since the word *stable* did not mean much to me, I wanted to know why David objected to my calling the soap film *strong*:

David: *You noticed that this bubble lasted longer?*
Maja: *Yes. That's because it is stronger. Now I can't say that any more.*
David: *Yes, but you have a* because *in there which I wouldn't have. I was just saying it lasts longer. I'm just describing a thing that means it's more stable.*
Maja: *I see; lasting longer means greater stability to you. It means that?*
David: *It's just the same thing. It's not an explanation of it.*
Sally: *He's defining stability.*
David: *It's not an explanation of anything, it's just a description...*
Maja: *It always means lasting longer?*
David: *Yes, it's always protection against shocks or disturbances. You say, "All that I observed about the film is that it lasts longer. I don't see that it's physically stronger in any other sense." Then you can go back and say, "Well, it* could *be possible that it was physically weaker, that is, easier to stretch, and yet lasts longer." There is no sense of contradiction there, whereas when you use the word* stronger *you seem to be inviting a contradiction.*

I finally accepted the word *stable* as describing the soap film but I was still not satisfied. It was a description, and I wanted an explanation. I had the funny feeling that stability carried an additional meaning to scientists. I wrote in my notes: "*Stable*: what does David know about it that I don't know?" Was there another meaning that I was missing or did David simply not want to go any further with the soap bubbles? This past week, when I was going over the transcripts and still finding myself confused and dissatisfied, I again asked David for an explanation.

This time he brought in molecules. Soap molecules, he said, are much more complicated than water molecules. Their two ends react differently to water. One end is hydrophilic and the other is hydrophobic. In a soap bubble, these molecules line themselves up in such a way that they act as stabilizers of the thin soap film. They help to keep the film at an even thickness while it is being stretched, and this keeps it from breaking.

I realize that I could have continued to ask, What exactly do the soap molecules do and why are they doing that? But just as Hedy was satisfied with the *description* of the water drop and the soap bubble, I am satisfied right now with the description of soap molecules, and I can accept the fact that their behavior accounts for the stability of the soap film without wanting to know any more.

Molecules have now become part of my thinking, although they are still very much on the periphery of my mental framework. Because it is easy to *talk* about molecules and atoms, to use the labels without understanding the concepts, David wants students of all ages to have a broad knowledge built on observation and experimentation before talking about the physical world in terms of atoms and molecules.* When we were studying heat and asking a lot of big questions about electromagnetic radiation, David told us that he would like to keep us in the eighteenth or nineteenth centuries for a while so we could arrive at an understanding of heat which paralleled the historical scientific development of that subject. We were too impatient; we wanted twentieth century answers even though we found them very hard to understand.

*How do you know when students are ready to think in terms of atoms and molecules? Timing is always one of the most difficult questions which teachers have to face. David, I think, prefers to err in the direction of being too late. Most science teaching errs grossly in the opposite direction.

Cubes, Bananas, and Plasticene

Since neither the soap froth nor the water drops raised the question of area/volume relationship in teachers' minds, David and Ron decided that the topic might be made more accessible if we spent some time working with wooden cubes, where the changing relationship could more easily be analyzed. This is how David introduced the cubes:

David: *How many faces does one of these little things have? (They were 3/4th" cubes.)*

Teachers: *Six.*

David: *Okay. Now, if you make the next bigger cube out of these little cubes, how do you do it? (Teachers make a cube that is two little cubes long, wide, and high.) Okay, there is the next bigger cube. How many cubes in that?*

Teachers: *Eight...four...oh, right, eight!*

David: *The first one was* one *cube with* six *faces. Let's give these faces a name. (Group discusses and settles on* minch.*) There are eight little cubes in this next bigger cube. It's two by two by two. Now, how many minches are there on the outer surfaces?*

Teacher: *Twenty-four.*

Teacher: *How do you get that?*

David: *You multiply, because you observe that there are six sides. It's still a cube and each side has four minches. Okay? Then, what's the next bigger cube?*

Although somebody correctly answered "twenty-seven," I asked David to slow down so teachers could take their time building this "next bigger cube." I sensed that several teachers were already quite confused.

Having been rather open-ended in the previous class, David was quite explicit in his instructions about how to make the cube grow, hoping that area/volume relationship would now become apparent. Little did he know what kind of trouble we would get into later because of his instructions to make "the next bigger cube."

Teachers reacted in many ways to this class. Hedy didn't like it. She hated the math. She reported having one fleeting insight -- "Oh, that's what it means to *cube* something, literally make a cube of it" -- but when she told someone about it she realized that

as soon as I said it I already couldn't remember or understand what I had just said.... I went on jotting down the numbers of cubes that would be in each succeeding size of cube but I was just multiplying. It didn't get any realer. I also tried figuring some ratios because I overheard Ron suggesting this to another person, but the ratios didn't mean much. So what? My usual feeling. Mostly it was an enormous strain to try to think about it so hard, so profitlessly. I went home with a headache, and I never have headaches!

Sandy, on the other hand, reacted in the opposite way:

I was completely delighted with that session.... The mathematics of surface and volumes was wonderful. I felt the full power of discovery as the relationships began to unfold before my eyes.

I love number patterns, too, and I was going to make a chart which would show every possible measurement of cubes. I had columns for length, n; area, n^2; surface area, $6n^2$; volume, n^3; and number of inside faces, $6n^2(n-1)$. David suggested that I add the inside to the outside faces and showed me how that becomes $6n^3$. I spent the whole class time writing down growth patterns in these columns and realized once again that there is no such thing in mathematics as a chart that shows *everything*. As you begin to see patterns and relationships, new ones keep appearing that can be added to your chart. I got so engrossed with all my number patterns that I completely forgot about the purpose of the class -- to explore area/volume relationships!

Sally had a similar experience:

Working with cubes was fun, but I became so absorbed with trying to find formulas that I totally digressed from what I was first interested in finding -- the ratio of surface area to volume. In fact, I wasn't thinking of the science work at all, I was just enjoying myself.

It was Ron who tried to bring us back on course and suggested that we try to find the ratio between the volume and the surface area growth of the cubes. What happens when the linear dimension of an object is doubled ("the next bigger cube")? We learned that the surface area of that object quadruples and the volume becomes eight times as big. (Area increases by the *square* of the length, volume by its *cube*.) For some teachers this was an exciting discovery; for others it was a source of great confusion. Myhra reported that after she got home

thinking I really knew what I was doing, I found out the next day that I had the formulas for figuring out area and volume reversed. It all seemed to make sense to me at the time. Now I'm not sure I really do understand what appeared so simple earlier.

When Ann wrote her weekly notes, she found herself

trying to make sense out of the relationships of volume to area. I feel I need to get out the cubes again. As the volume gets larger, the area gets smaller because there is more space for the faces of the cubes to be hidden in the interior of the cube. Is that right?

Sandy, on the other hand, found these relationships "crystal clear," but added, "What is not so clear is what this has to do with nature." Shelley also caught on to the relationship between area and volume but, like Sandy, she went home "still looking for what all of this meant in 'real life.'"

When David decided to use the wooden cubes, he was not planning to make an analogy between their inside faces and the interior soap film which increases so rapidly as you beat the soapy water. But the mention of inside faces during the class made some people think that there was meant to be a connection. Mary Jane asked: "If there is any correlation between surfaces of soap bubbles fortifying themselves when they come together, are these molecular fusions of a different quality than wooden cubes which don't adhere to one another?" I had made the same observation: the more little cubes you used to build bigger and bigger cubes, the more wobbly the whole thing became. "It does not get stronger like the soap film," I said to David, "it falls apart." Somehow, I thought that the inside cube faces were supposed to illustrate why the soap froth gets stronger as you keep subdividing the interior film by beating it. The inside cube faces *did* come up for discussion in class, but not for that reason. It was found that you could set up another ratio of *their* growth in relation to either volume or surface area growth. (I didn't realize that until writing this.)

The next day, at our weekly staff meeting, Ron expressed surprise that teachers who understood how a cube *grew* had a hard time figuring out how volume and area changed if the process was reversed and the cube was made to *shrink*. That didn't surprise me in the least.

I could see why Ron might think that if you could do the calculation in one direction the reverse would be obvious, but to me -- and to most of the teachers -- it seemed like an entirely new problem. In fact, I got quite confused when David tried to help me understand this: "Pretend you have a nice big cube," he said, "and you saw it right there in the middle, this way. Now you don't have a cube anymore. Now put the pieces back together in a vise and saw them in the other direction...."

Maja: *Then you have four cubes.*
David: *No, no, no, you have four pieces that are twice as long in one direction.*
Maja: *You mean with two cuts you don't get four cubes?*
David: *No. (He describes how you have to cut the cube to get back to smaller cubes.)*
Maja: *So you have to have three cuts?*
David: *Yes, three cuts. How many cubes will you get out of this?*
Maja: *I suppose eight.*
David: *You suppose? What do you mean you suppose?*
Maja: *I'm saying that because I know the number from working with my growth patterns but if I didn't know that and had to think of the actual cubes, I'm not sure I could figure it out.*

I don't think I ever visualized a cube when people talked about cubing a number. I related "the little two" to a geometric square; the "little three" had remained an abstraction. I now became intrigued by these geometric representations of powers and wondered whether perhaps there were other shapes, such as a tetrahedron, which represented powers past three. David said that my analogy was right but that in the physical world you can't go past three without artificial inventions. You can go to higher powers but you don't have any geometrical representations for them because space is three dimensional. In the real world, there are only three dimensions: east/west, north/south, and up/down.

When David said this, something clicked in my mind. The cube suddenly became a real-world, three-dimensional object which therefore *had* to grow in three directions. If you increased the length of the cube from 1 to 2, you also had to increase its width from 1 to 2 and its height from 1 to 2: That's how you got 2 x 2 x 2, or 2^3. I had known the formula for many years but I had not grasped the logic of it. Now I can say, *obviously* the volume of a cube (or of any object, I was to learn later in the course) growing in three-dimensional space increases faster than the two-dimensional area or the one-dimensional length. Why had I never realized this before? Even though I had played with squares and cubes of different sizes to get the visual picture of these growth rates, the real meaning of these relationships had escaped me. Now I have crossed this barrier and have arrived at a new plateau in my understanding. What I still lack, however, is fluency with this new idea, a fluency which will come only from experience in everyday life and from much thinking about size and scale in the physical world.

I had another insight that day which I recorded in my notes: "If there is this relationship between linear area and volume growth, then everything that has size must be affected by it. Since everything in the world *has* size, does that mean then that everything in the world is governed by this law?" No wonder David keeps coming back to this topic as providing us with a powerful organizing idea.

David commented on the many difficulties that had arisen in the cube class:

I have this hypothesis that the notions of large and small all have these conventions in our mind long before we learn anything about numbers. It's a mixture of all these things we are talking about: length, area, and volume, a mixture of things that are not distinguished from each other by common sense. They are just taken intuitively as big and small, so if something is twice as big in one sense, it can't be four times as big in another sense, because there is only one sense of big *that you recognize consciously...the commonsense idea is an undiscriminated mixture of two or three ideas which the scientist or the mathematician wants us to sort out and use independently of each other. And we can't do that until we become reflective about it and see the need in terms of some interest of our own, some curiosity of our own. Does that make sense?*

It made a lot of sense to me. I had puzzled about this problem in a slightly different way many years earlier when I was first introduced to the topic of size and scale. I had been playing around with squaring and cubing different numbers to get a feeling for area and volume. I suddenly became concerned that the same number symbols were used to describe very different kinds of measurements. I had written in my notes:

2^2 equals 4. 2^3 equals 8. Four refers to an area *enclosed by four lines of two units each, but what does the four really stand for? How can a length and an area, which are so different, be described by the same symbol? The cubic measurements are even more difficult to comprehend. The fact that the "4" is followed by "square feet" and the "8" by "cubic feet" doesn't seem to make enough of a difference. These are just words without associations or meaning.*

The class brought me back to my confusion about the meaning of the words *area* and *surface area*. In the past, when David had talked about the surface area of leaves on a tree, it had never occurred to me that in *his* mind the surface, which to me implied only the top boundary -- like the surface of a pond -- would include the entire outer layer of an object. For leaves, it would include top *and* bottom; for a table, the top surface, the underneath of that top, the edges, as well as the table legs. In *my* mind, area had always been flat, facing in just one direction. When I heard the term *surface area*, and couldn't visualize it as flat (as with the root hairs mentioned in our first meeting), I became confused. The surface area of the cube included all six of its faces and that was difficult for me to incorporate into my notion of area. This notion was a mixture of high school geometry -- rectangles, triangles, circles, always drawn on a flat piece of paper -- and area as it comes up in my everyday life -- a floor to be carpeted, a wall to be painted, a lawn to be fertilized. Even in real life, area was always flat.*

*When I use my common sense, however, and don't think about what area is supposed to mean, I know that if I want to cover the area of a table with paint I have to buy enough paint to cover the entire outer surface and not just the table top!

Because my confusions are often indicators of similar confusions among the teachers, we planned to devote the next class to further explorations of area. "In the physical world," David said, "area is always the area of a real surface." We were going to get real-world objects with easily removable surfaces -- fruits and vegetables which could be peeled. Then, David said, we could transform the curvy outside surfaces of irregularly shaped fruits and vegetables into something that is represented on a flat piece of paper as the *area* of geometry texts.

In class, before working with our edible materials, David asked the group, "How do you think about the area of that table over there?"

Ann: *I think of just the top of it. When you talk about surface area, it would be all the exterior you see.*

Maja: *Before this class, what was your image of area?*

Myhra: *Maybe a rug, an area rug.*

David: *And how do you specify the size of such a rug?*

Teachers: *In square feet...by the exterior beneath it...eight by ten.*

Myhra: *You're buying a rug to fill a space and you give the dimensions, like eight by ten, so when they come to sell the merchandise you know whether it will fit or not. As opposed to, if you know how many square feet you had in your living room and you go to look for that number of square feet in your rug.*

David: *It might not fit.*

Sally: *It might be the right number of square feet but a different shape.*

Myhra: *Yes, you want to know how much contact the rug is going to make with all the surface.*

Shelley: *Does the rug have two surface areas? Do you count the top and the bottom?*

David: *You see how really complicated this turns out to be?*
Sally: *And when you're talking about acreage outside, in the field, that's different too. It's not just flat, it's up and down.*
David: *That's right, but you sort of treat it as flat, don't you?*

By raising questions about area, by accepting our way of thinking, and by admitting that he hadn't really thought much about the difference between area and surface area -- "I've never been conscious of either using the word *surface* or not, so it's very useful to me to realize that that caused trouble" -- David got us to think much more deeply about the meaning of area. We wondered whether both sides of a surface should be counted, and how a hilly piece of land was measured. Then Ron challenged us even further by asking what the area of a square foot of velvet would be -- "Is it a square foot or the area of all those little hairs?"*

The fruits and vegetables were invitingly arranged on a table when the teachers arrived for class. "These are friendly forms," wrote Hedy about her initial reaction when walking into the room. "No matter what we do with them, I will like this." Everyone seemed to feel the same way. As soon as the teachers had peeled the fruit, they asked for graph paper. Initially we tried to withhold it,** but Sandy was insistent: "How can you find out the surface of a banana skin without graph paper? You have to have a unit to give an area measurement." So I handed it out. We probably didn't stress sufficiently the point about comparing surfaces. When Mary Jane wondered whether she could find the surface area of a zucchini by wrapping a string around it, we should have encouraged her to try it and then try to wrap other fruits, or different-sized zucchinis, to get comparative amounts of string. At the end of the class, David mentioned that the area occupied by a peeled orange could be covered with rice grains and, when this was done with areas of other fruits, the quantities of grains could then be compared. Perhaps we should have mentioned such possibilities to the teachers earlier, to keep them away from graph paper.

Most teachers peeled different fruits and then laid the peels out on graph paper to determine area. They enjoyed the work but again, several people wondered why they were doing this. Mary wrote that she was impressed to learn "that seemingly compact shapes could have a significant surface area," but she wondered why one would ever want to know how much surface area an apple had: "Isn't there an easier way to puzzle it out than peeling and laying it out, or peeling and weighing it? What value is this knowledge to me?"

We had also put out balances so that teachers could get at surface measurements by weighing sections of skin and setting up ratios. The weighing was a big

*As I write this, I begin to wonder what is included in the measurements of large areas of land, like national forests? Do they measure only the surface of the land itself, or do they include everything that grows on it or protrudes, like trees, rocks, and mountains? I do know that surveyors wouldn't include the surface area of all the needles in a forest of evergreens, but I'm having fun thinking in this new way.

**At our staff meeting, we had wondered whether to put out graph paper for the work with surface area. David would have preferred teachers to approach these explorations by *comparing* different surface areas without immediately going into numerical measurements. "I'm a little bit nervous about prematurely dividing something up into squares," he said. "I think there is some intuition of quantity that doesn't have to be translated right away into numbers."

success, but the idea of setting up ratios caused considerable trouble. Myhra wrote: "Why did I have to weigh a square inch? Couldn't I have weighed a quarter of that? Is the square inch part of the formula? Does it make any difference to use inches and grams in the weighing method?" I know from my own experience that ratios present a big barrier. It took me a long time to understand that a ratio is a relationship of numbers and that the numbers can be quite different but still have the same ratio. It is not easily accessible knowledge.

I suggested to Shelley that we try weighing the area of her palm. She drew around her hand on a piece of cardboard and cut out the imprint. She then made some units out of the same cardboard -- squares which had areas of 1, 4, 9, and 16 square inches. She weighed the cutout of her hand on an equal-arm balance and found that it balanced with the two- and three-inch squares. Now, what was the area of her hand? First we thought it was five square inches, but we quickly realized that this was wrong. Then we figured that we had a square of nine square inches and one of four square inches. Could we add these together? We thought the answer might be thirteen but we really weren't sure if square inches could be added.

When I told David about our problem he said, "You could have cut the hand up into two pieces and you would have known that you could add them to make the hand again. You could have cut up your squares into nine and four smaller squares and then mixed all these squares up together and you would have known that you had thirteen." How obvious, I thought. Why were we so confused? Here was a good example of using numbers without having a real understanding of what they mean. Then David explained:

Add *doesn't just mean* arithmetic *add, it means physically put together. You can arithmetic-add because you can do the other. When a child adds two handfuls of pebbles together, that is adding. We get the arithmetical meaning of the word from that, but it wouldn't mean anything if somewhere in the background we didn't physically put things together.... Amounts of surface can be added and divided. You think of division not the way you think of arithmetic but the way you think of scissors and you think of adding as moving two pieces together or rearranging them.... You can add areas arithmetically but you can also put them together and see that they make an area twice as big. The meaning of* add *and* subtract *and* divide *that a child knows with the physical operations and not with numbers is the meaning you need to recover here.*

We have moved so far away from these original meanings that we couldn't solve the simple problem of adding nine and four square inches!

If we had a feeling for area, the little puzzle of whether four square miles is the same as four miles square would also be easy to answer. Instead, many of us probably felt like Hedy who, after having the difference explained, exclaimed, "That's incredible! I had no conception that that would be different. The words don't give you a clue. Having *square* and *mile* and a number in the same sentence -- they could be in any order and they would all sound the same."

Most people worked only with the concept of area in this class, but some teachers got into volume -- another big stumbling block. Sandy reported:

I understood when I worked with the cubes how to get from length to the total surface area and the total volume. Then I thought there must be a way to just simply measure the banana and from that length measurement get a total surface area and total volume, using that formula. I was wishing it would be so easy but with the banana, there is a strange number there and it's not clear and I don't know what it is or how to find it.

Sandy's problem brought us closer to the question of whether the relationship between volume and surface was peculiar to the cubes or whether it held for all shapes. "That really is the big step," said David, "and that is the step that almost never is taken anywhere in school." He then talked about exploring surface/volume relationships in a different way: Could we change the shape of things and have the area stay the same? What would happen to the volume? Would it stay the same or would it change even though the area remained constant? Could we compare a whole lot of things of different shape that have the same amount of surface? What will be the difference in the volume? Which shapes have the most and which the least volume? These were new questions which most of us hadn't even thought of.

At our next staff meeting we decided to get a lot of Plasticine so teachers could make different-shaped objects and then change their shapes to study area/volume relationships one more time. We also thought Plasticine would be useful for those teachers who still had difficulties understanding what happens when you make a cube smaller. It would be easier to cut up a Plasticine cube than one made out of wood.

Like every other class so far, the Plasticine class revealed additional confusions. A number of people worked with Ron, changing the shape of a piece of Plasticine from cube to sphere to pancake to snake, observing how the surface area changed while the volume remained the same. Others got interested in a question that Marsha asked: "Suppose I have an apple and I want twice as much apple?" Any child would probably know how to get "twice as much apple," but we suddenly became confused! In the examples given us in class, the linear dimensions were doubled, so the volume was always eight times larger. We now wondered what you had to do to get just double the volume. Marsha decided to work on this question and started to make a Plasticine apple. Several teachers decided to make Plasticine cubes and then try to double their volumes. First they made two cubes of the same size, then they squashed these two cubes together to make one larger cube. Although we all knew this new cube to be twice the volume of the original cube, we didn't think it looked twice as big. Myhra suggested we make three cubes of the same size so that after the double-volume cube was made, we could compare it with the original size. Everyone was surprised, and somewhat disbelieving, that the double-volume cube looked so small. I believe all our thinking about how cubes grow went back to the class in which David introduced the topic by telling us to make "the next bigger cube" out of 3/4th inch wooden cubes, where *next bigger* meant doubling the length and, therefore, getting eight times the volume. We completely forgot that in real life there could be an infinite number of in-between cubes. The volume of cubes grows dramatically when you double the linear dimension, but obviously cubes can grow at any rate you choose.

The relatively small double-volume cube led to a lengthy discussion of size: What does *doubling* something really mean? Do you always have to specify what dimensions you are doubling? Do people who are familiar with this concept just automatically think of area as being four times as large and the volume eight times as large when the linear dimension is doubled? We realized that we had never made these distinctions when talking about size, and we also realized that our intuition of volume was rather undeveloped.

A few teachers wanted to see what would happen when you cut a cube in half. They made a large Plasticine cube, the same size as a three-unit cube (the equivalent of twenty-seven little cubes), and then cut it in half -- one cut in each of the three directions. They expected the resulting eight smaller cubes to be the size of the wooden two-unit (eight piece) cube. Why, they asked, were their Plasticine cubes smaller? Common sense would tell you that half of three is one-and-a-half, not two, but since we had always made the wooden cubes grow by units of one, a one-and-a-half unit cube didn't fit the model.

David later said that it had probably been a mistake to use the wooden cubes to introduce the idea of three-dimensional growth. "We have been going in multiples of that unit, and that is nonessential to the idea." It may have been nonessential to the idea but it is where most of us got stuck in our thinking because we didn't yet fully understand the idea, nor did we know where it was supposed to be leading us. Hedy wrote, "Part of my lack of interest in these size topics is that they are so abstract. Cubes are really meaningless to me. They explain something to somebody, but I haven't yet asked the question that they are supposed to answer."

Another rather extraordinary confusion arose in this class. Sally wanted to show that cubes of *any* size would grow in the same proportion as the 3/4th inch cube we had been using. She took a set of Cuisenaire cubes, where the smallest unit is one cubic centimeter, and stacked them up to show how their volume changed each time the length was increased by one unit. Several teachers (including myself) were surprised that the size of the Cuisenaire cubes didn't increase as rapidly as the 3/4th inch cubes. It took us a while to figure out that the cubes grew *in proportion* to the original unit, and that this *proportion* remained the same for all cubes, although their actual sizes could be different. This insight led to another interesting discussion about the meaning of *one*. If the number one cube can be of any size, then what does *one* really mean? Mary wanted to know. How can *one* describe a cubic centimeter *and* a cubic inch cube?*

*I often wonder why we fail to use our common sense when we are learning something completely new. Our confusions over size reminded me of the problems of traditional teachers who sometimes have a difficult period of transition when they want to change their teaching approach. They tend to give up all their useful traditional teaching skills as soon as they start trying new approaches, as if the old ways and the new couldn't be combined. Similarly, when we are confronted with new science concepts which we don't yet fully understand, we don't seem to use *any* of our earlier thinking, even though a lot of it could still help us with questions and confusions.

It makes good sense that, after our exposure to length, area, and volume growth, we would start to wonder what *big* really means. Our older, generalized notion of size had to be refined so that we would differentiate between *longer* or *heavier* or *having more area* or *more bulk*. On the other hand, it doesn't make sense that we should think about growth in terms of "the next bigger cube" or that we would expect half of three to be two, or that we should think a cube made of twenty-seven one centimeter cubes would be the same size as one made up of twenty-seven 3/4th-inch cubes. We certainly wouldn't expect a mouse and a dog and a horse to be the same size if we were told that they all doubled their volume or weight in the first three months after birth!

Most of us got confused all over again when David and Ron switched from saying "when you double the length, the area is four times as big and the volume eight times as big," and talked instead about area being the length squared and volume being the length cubed. Because we learned this rule with the specific example of doubling, we took the example to be the rule. Myhra wrote in her notes, "I'm trying to remember now that the 3rd power is cubing and volume is always cubed. Can I say: when you cube something, it is eight times as much? I think so."

I don't remember the Plasticine class as being frustrating. We laughed a lot about our troubles and we thought a lot about what we were learning. Shelley, though, must have hit a real stumbling block when she started to explore volume with the Plasticine. She wrote:

People went in different directions. Some seemed to know with confidence what they were doing and others (like me) watched, looking for a place to start. I *felt the same despairing frustration I always feel when people start to apply formulas from their memory and say "just do this...it's simple, really." We're all at such different levels of experience.*

Size and Scale in the Real World

Sally and I were fascinated when David described how the digestive system and the breathing apparatus change from simple one-cell creatures to animals the size of an elephant. He talked to us about "the fundamental biological fact that living things have to maintain roughly the same area of exposure to their air and food supply as the small things do that get it through their outer surface.... The ratio of surface to volume or surface to mass remains constant." He then explained to us how food diffuses through the surface skin of an organism "so that the amount that can get through is limited by the amount of surface area, whereas the tissue to be fed is three-dimensional. It's a very basic fact that the architecture of living things is accommodated to this and that's why little things are different in shape than big things."

After several teachers had mixed questions about Julian Huxley's essay *The Size of Living Things* which we had been given at a previous class, Sally suggested we leave our mathematical struggles for a while and devote a class to talking about the biological implications of Size and Scale, which Ron agreed to lead at our next class meeting. Space doesn't allow me to report the details of the teachers' confusions, which prevented Ron from getting to his topic until almost the end of the class.

At our staff meeting the following day, we discussed the teachers' questions about volume. As usual, I shared many of their confusions. First, I found out that I rarely ever thought about the volume of an object. If I wanted to describe how a cube grows I would just say that it got bigger "all around." I wouldn't say that its volume had increased. I realized that I didn't even think of volume as a unit of measurement! I was also troubled by the fact that *volume* could refer to a solid object and at the same time to an empty space. For some reason, I resisted thinking of volume in this double way. In class I had asked, "How are you *supposed* to think about volume? Is there one way I should think about it that is more correct than another? How do most people think about volume -- as something empty or hollow or as a hunk of something." David tried to explain the two concepts to me: volume as capacity -- "a container that defines or surrounds a certain piece of space, a set of walls within which you can trap something" -- and the other idea which he called *bulk* -- something that takes up a certain amount of room. "There are two ideas of volume that have to be connected: one is a container that has a certain capacity and the other is what fills the container." David wondered why we had trouble connecting these two ideas in our minds, since they were so closely related. Right now, I am wondering the same thing. Having made the connection and assimilated the double way of looking at volume, I cannot remember what troubled me last year.

I also had a hard time thinking of something spread out thinly, like paint on a wall, as having volume. I know that paint has volume when it is in a gallon can but once the paint is on the wall it loses its three-dimensionality for me and seems to become part of the area of the wall.

To help clarify some of these confusions, we planned to assemble a larger variety of containers of different sizes and shapes for the next class, as well as salt, rice and beans, and paper to make cones and cylinders.

Sally then reminded David that at a previous staff discussion we had talked about the fact that *all* shapes, not just the cubes we had worked with, follow the same growth laws. This came up when David tried to help me understand why there couldn't be a mountain one hundred miles high. "The base would have to support more and more weight for every unit of surface and if the mountain gets too high, the rock will bend and the earth will begin to act like a liquid.... The crust of the earth will be soft from these enormous pressures."

Sally and I had been very excited by this new information which we were sure the teachers did not know. Sandy had been asking in her notes, "What happens to the growth when an object is irregularly shaped? Natural objects are not cubic but are asymmetric and uneven and I don't see how one can even make a linear measurement on most real-world things." Sandy's question, and other confusions, made it clear that this idea had not been adequately dealt with. We hoped that when the teachers worked with the materials we were preparing they would be able to extend their understanding from cubes to other shapes.

David believes that there is a time in people's learning when theory can help pull things together. Before our work with volume, he wanted to talk one more time about the relationships among length, area, and volume. He hoped that we would be able to go beyond numbers and become comfortable with the knowledge that if you change any one of these measures the others will change in a constant ratio.

In class, David started out by saying that he wanted to get away from the cubes, which allowed us to increase things only by fixed amounts: "We really need to get away from fixed units that you count. Think instead of length and area and volume as quantities that can change by arbitrary amounts."

David then defined length, or the linear dimension, as the distance between any two fixed points on a shape:

"If it's an elephant, it could be the distance between the tip of his tusk and his tail, or the distance between the bottom of his feet and the top of his back... you can pick anything as long as you pick the same length on the small elephant and on the big one." Area, David continued, could be the bottom of the elephant's foot or the surface area of his trunk or "all of his skin area all the way around." Sally thought it was neat that you didn't have to think about his entire surface but just one part of it. "If you preserve the absolute similarity as you scale the elephant up or down, it doesn't matter which, you can get away from any worry about whether you count the area on the bottom of his feet. You can, but you don't have to," said David. He did not say much about the third quantity, volume, except that we would spend some time looking at it in class. And then he gave us "three simple sentences:"

1. The area is proportional to the square of the linear dimension.
2. The volume is proportional to the cube of the linear dimension.
3. The ratio of the volume measure to the surface measure is proportional to the linear measure. (David said that this statement was "the big one, the clincher," but that we shouldn't feel we had to understand it right away.)

The main thing to remember was that if you scale an object up or down, if you make it larger or smaller but keep the same shape, "the length or distance and the area and volume all increase or decrease together, but not in the same proportion...there is a fixed relationship between these three quantities so long as you keep the shape absolutely the same."

There were many questions during David's talk and we didn't have a great deal of time to work with the volume materials we had prepared. When we started, however, it quickly became clear that our intuitions about volume were quite poor. I found it hard to believe that a cube ten centimeters on each side would hold as much liquid as a one liter bottle. Marsha related how she had ordered an end loader of dirt: "I thought I wanted two until I transferred the one load with a wheelbarrow down to my garden. I couldn't believe how much it was." And Sally told how she ordered ten tons of gravel: "I figured out mathematically how many square feet I was going to cover by what they told me, and when they dumped it I said, 'That little pile?' I thought I had gotten gypped until I started to spread it out."

We have little experience measuring volume in everyday life, except using standard capacity measures such as cups, pints, or quarts. I wonder, therefore, whether we might be judging the "amount of stuff" by some linear dimension. Maybe we are just looking at the length or the height of a pile of stuff, or even at the length *and* height, but somehow, as Sally said, "we are not visualizing those three dimensions." We obviously need to devote more time to volume.

Toward the end of the class, I asked David if he could spend a few minutes talking about where our new understanding of scaling would take us if we gained more fluency with it. He said:

For Ron, it would be the fact that you cannot scale living things while keeping their form exactly the same. The only way you can succeed in getting bigger living things is to change their form. It's the nonscaling involved in living things that's a fascinating and very rich topic. For me, in a much wider sphere of application, it has to do with the way the properties of things change with size in general, not just for living things. Why little drops of water would sit on a wax paper and make all those perfect little spheres while big globs of water will flatten out and make almost flat surfaces. Why things that are bigger than the planet Jupiter are fiery hot and why things smaller than the planet Jupiter are apt to be fairly cold. Why things bigger than the moon are always round. If these things were well learned and acceptable in the imagination, they would give you a kind of classification system for all the furniture that has been discovered to exist in the natural world from atoms to galaxies.

Sally wanted to know why things that are bigger than Jupiter are fiery hot.

Things that are bigger than Jupiter get squeezed together so much by their gravitational pull that the atoms that can maintain themselves in the cool state get crushed and that leads to nuclear reactions. Hot, hot, hot stuff. What's the biggest thing that can be shaped like this tomato can? If it gets bigger and bigger, that gravitational pull is going to dominate finally and it's going to squeeze it together and make it more like a sphere. And that goes back to Ron's example of how smooth the earth is. Mt. Everest's height is only a tiny, tiny little fraction of the four thousand miles which is the earth's radius....

Why is this important? It's a very unifying thing. It's not detailed, it doesn't tell you a lot, but it gives you a kind of framework. You can say, Gee, for anything that's as small as a drop of water sitting on wax paper, it's the contraction of its surface skin that is going to be the dominant force, so it's going to want to shrink into a sphere. When you get a much bigger drop of water and put it on wax paper, the dominant force is weight, gravity, and so it's flattened out. There are lots and lots of changes you can observe in the kinds of things that exist, which are of necessity subservient to this principle. Like the little tiny *animal which has to burn food mostly just to keep warm because the surface area through which he can lose heat is so big in comparison to this weight, whereas a great big animal uses a much smaller fraction of his weight and energy to keep warm. There are just endless examples. You can't list them all, but once you get into the habit of thinking in these terms you begin to notice.*

That was a big challenge. Sally and I had supper together after class and we talked about some of the things David had said. "Perhaps it's okay not to understand everything," said Sally. "It keeps you wanting to know more."

In the last two classes, teachers used all the materials we had collected for work with volume, trying to clarify questions about the relationships which David had explained. Shelley spent one class working with Sally and Harriet on the volumes of cylinders and cones. She wrote, "I was finally doing what I really wanted to do back in high school geometry. I had so much trouble with solid geometry and the sad part is that I don't think I ever even handled, much less investigated, a cone, sphere, etc. They were always drawings. So it was very exciting to discover the volume of a cone on our own."

After the last Size and Scale class, Shelley reported:

We decided to start making increasingly larger cones. I wondered how we could be certain that one cone would actually be double the size of the previous one. As we started, I saw that if one linear measurement was increased a certain amount, then all the linear measurements would increase by that much as long as the same shape was retained. I don't know why I never saw that before -- it's so obvious now.... We saw that if the linear measure was 2X the original, then the volume was 2^3 *and if the linear was 3X the original, the volume was* 3^3*, etc., so we predicted our next numbers and were correct. Amazing, what a good feeling!*

It had taken Shelley and most of the other teachers over half the semester to grasp the relationship of length, area, and volume. We were now at the threshold of understanding the significance of scale in the natural and physical world. Though we have a long way to go before we can easily apply this new understanding, the first big barrier had been surmounted.

3

*The Research Project: Learning About Light and Color**

*This chapter, by Maja Apelman, appeared in somewhat different form in OUTLOOK, No. 53, Autumn 1984.

At the end of the first semester, I interviewed the participating teachers and asked them, among other things, if they would be interested in continuing for another semester and, if so, what topics they would like to explore. Many of the suggestions had to do with light, color, optics, and related topics.** After many staff discussions in which we considered the teachers' suggestions as well as a number of additional subjects, we finally settled on *Light and Color*.

**We had touched on electro-magnetic radiation at the end of the Heat classes and most of the teachers wanted to spend more time on this subject.

At the first orientation meeting (some teachers had not yet decided whether to attend) David discussed the goals of the research. He spoke of the need to reorganize "the furniture of the mind when a new idea has to be accommodated to an old way of thinking that no longer works:"

This is not just a matter of getting rid of an old notion, it involves restructuring.... Instead of saying that common sense is wrong and science is right, say common sense is right too, but it's a different representation using a different language useful for somewhat different purposes. We don't want you to discard common sense but we hope you will try to acquire other ways of thinking which you can relate to common sense.... We want to explore the relations between our common sense understanding of the world and what the scientific world considers to be elementary, scientific ideas that are important and powerful.

The teachers were asked to bring in notes describing how they thought about light and color. "We are interested," Hawkins said, "not in whether people's ideas are right or wrong, but in what their ideas are. We are trying to find out *how people think*." Here are excerpts from what the teacher wrote:

I never thought of light as being anything more or less than a whole. Light was always there, or it wasn't. I was amazed to find out a few years ago that light can be broken down into colors. Light seems to be what allows colors to be seen. (Betty)

What is light? I don't know. I see it. It can be bright. The brighter it is the more light there is.

Does it travel? I don't think of it as something that moves. I think of it as just being there wherever it is and there is more or less of it. There's enough for me to see by or not. It can be dim. Like more or less moisture in the air perhaps. I am told of course that it travels but I don't experience it that way...I wouldn't offhand think light and color were the same topic. Light is light and colors are colors. You wouldn't have colors if light didn't light them up to be seen but not that colors are *light. And yet there is a rainbow.* (Hedy)

What exactly is a light year? Is dark the opposite of light or is it just a gradation of light? Do prisms separate the color inherent in light or is it the glass that contains the color? Light colors mix differently than paints. Why? How are rainbows caused? What is irridescence? (Polly)

What makes fire flies glow? What makes rainbows? What gives them color? What makes lightbulbs work? How do prisms work? Why can you see sunrays in a sunset and not at other times? How can mirrors reflecting sunlight burn a hole? How does a reflector on a bicycle work? Is light a particle or a wave? Why does light look wavey in water? or change shape? How do colors work? Is it true that light travels at a certain speed? If we lived underground would we lose our sight like moles do? Why do some things reflect light (water, sand, cement, etc.) and other things do not? Or does everything? (Marilyn)

To describe *my* very beginning thoughts about this subject, I must go back to notes from an undergraduate course on energy, which I audited as part of my research assignment. Here are some excerpts:

We've gotten into radiant energy in class and that gets us right smack into light waves--a subject I have avoided for many years. Although I knew that I would have to tackle it one day, because I often got into topics which required that understanding, I was never quite ready to make the effort. The reversed image in the pinhole camera, the round sunspots in the shadow under the large elm tree, the colors in a prism, rainbows -- these and many other experiences touched on the subject of light. Yet whenever I looked at the sun, a lamp or whatever -- I would retreat. "Some day I will have to learn it," I said to myself, "but I'm not up to it right now."

Now I have *to learn it. And I'm probably more ready than before since I have at least some awareness of light rays. They aren't a complete shocking sort of surprise, forcing me to rearrange comfortable existing knowledge. My "naive" understanding has been jolted many times before...*

Greenhouse effect -- locked cars in the summer: Why do I have to think of light *as being changed to heat* *inside a car? Why can't I just think of the* heat *from the hot sun as getting into the car and because the windows are closed it stays in there? I never really wondered before how the heat gets into the car: everything that is closed and stands in the sun gets hot!...*

Absorption and reflection -- they are opposites, though it is hard to understand how something you can't see gets absorbed. I can see light being reflected; I can't see light being absorbed. The substance that absorbs it doesn't get lighter; it gets warmer. There is transparency -- light can go through some things but not through everything; and different wavelengths of light go through different things. Which gets me to the spectrum and wavelengths and frequencies. Somewhere in there is flow: energy flow, heat flow. Do they talk of light flowing too or only heat?

In the energy course I tried to understand the relation between heat and light. In my first notes for the course on light I wondered about the relation between light and color: would I ever have made that connection, I wrote, if I hadn't attended the planning sessions? I certainly never asked myself why things were of a certain color. I decided to make a list of things I knew about light:

--*Light travels in straight lines* (whatever that means)
--*Light gets bent when it passes through certain things* -- water? *Bent* I think is a bad word. It implies to me something straight that becomes rounded. But that's not true. It just means it turns a corner with a sharp angle -- but it still goes in a straight line afterwards.
--*Light -- white -- is a mixture of all the colors of the spectrum.* That has always been somewhat of a mystery to me. But one learns to repeat the words. Perhaps it is not totally useless to have some of these barely-understood science tidbits. Then when you do begin to understand something, you can connect it with these statements and say -- "Oh, that's what this means! I see."
--*Light is related to heat.* It can be changed to heat, inside a car, that's the famous greenhouse effect.

After making the list, I started to raise some questions:

What is light anyway? How can you talk about something so abstract? How can you describe something that you can't see (well, not really), touch, smell, etc.? I guess you do see light, or rather you need light to see. So light becomes related to vision. Light plus eyes makes vision possible. If you need light and eyes to see, what do you see? The object which reflects light or the image on the retina?

David had said, "Light is emitted by excited atoms." How do excited atoms behave? And how do you know they are excited? Because they can't emit light? How did people ever figure that out and how is that motion different from the motion of a molecule which emits heat?

If color depends on wavelength, does light come through the atmosphere in separate colors? Then how is it put together to become white light? Do we ever see white light since most things have color?

These extensive quotes from the teachers' notes and my own* explain, better than summaries could, the kind of thinking -- tentative, speculative, circuitous -- which occurs when a new topic or idea is introduced. The rush of questions seems to be triggered by the intensive effort to assimilate new ideas and by the sudden realization that things which have never been understood before may actually become accessible.

*I tried hard in the Light course to keep track of my thinking and learning. I find that in writing things down, my thinking becomes more focused, as unexpected questions and answers pop into my mind. I have found, however, that it is extremely difficult to keep track of all the thoughts that relate to the learning of a new concept. I would virtually have to walk around with a tape recorder so I could talk into it whenever I have an insight, a question, or a confusion. Thoughts come into my mind at quite unpredictable times and it is impossible to remember the whole learning progression. Keeping detailed notes along the way helps to record the stages in understanding which otherwise are quickly forgotten.

Experiments with Light and Color

Mixing Light: Colored Shadows

"We'll get into trouble if we start with color," David had said in one of our planning meetings. For a number of reasons we ended up starting with color -- and there was plenty of trouble!

We had set up three working stations, each one with three projectors and with a number of red, blue, yellow, and green colored gels. The projectors were arranged so that the beams of light overlapped on the wall. The lesson was to be on additive color mixing.

We worked in three groups, each one with a staff member as leader. We mixed light and most of us were startled by the unexpected results -- unexpected because our point of reference was that of mixing pigments. Even more startling, however, were the shadows we produced with hands, sticks, and other objects. I found these multicolored shadows very confusing and wondered if I had ever seen colored shadows before. "How could a shadow be anything but black?" I asked, and then I wondered:

If a shadow means there is no light hitting this area and the sun or a lamp make a black shadow, why do colored gels make colored shadows? What colors make what shadows? When you have several lights and gels, why and where do you get black and why do you get different intensities of colors?

Some of the teachers asked similar questions:

What causes the shadows to be the colors they are? When you shine three lights it never made sense to me what *color the shadows were going to be. I never knew what colors would appear as a result of the color combination of light.* (Cindy)

If a normal grey or black shadow is the absence of light, then what is a red shadow the "absence of?" Something is being let through and something isn't. Why does a gel create a colored shadow? It's still light being obstructed -- why doesn't it leave a grey shadow? Why does the color come through? (Hedy)

I wanted to get the shadow problem solved and decided to work it through slowly and systematically. I set up two projectors, got some colored gels, and asked a friend to work with me. First we turned on one projector and made a shadow with a yardstick. It looked black. Then we turned on the other projector and got two grey shadows. We figured out which projector made which shadow but we didn't yet understand why the two shadows were not as dark as the first shadow made by only one projector. Now we placed a red gel over one of the projector lights, still holding the yardstick in front of the area on the wall where the two projected beams of light overlapped. The previously black shadow became bright red. I was puzzled: How could red *cover* this deep black? I wondered. Then I remembered: Black is the *absence* of light. The red can *move in* because there is *no* light there. The black shadow is caused by the yardstick held in the path of the light of the first projector; the second projector lights up the shaded area with its red light. When all your experience with color is related to pigment, this is a difficult switch to make.

It took a good deal of time and much hard thinking to figure all this out. Ron had wondered earlier why none of the teachers had caught on to the geometry of the projection and the shadows. I don't think he realized how long it takes just to become comfortable with something as new and strange as colored double and triple shadows, some with complementary colors which aren't even on the wall! You are so confused at first that it takes a while to realize that geometry is involved at all.

Several classes later, there were renewed questions about the colored shadows. "If you covered one projector with a green gel and one with a red gel, why is the shadow cast by the green light red and the shadow cast by the red light green?" someone wanted to know. David replied:

I think a lot of people have real trouble seeing which of the shadows is going to be red and which is going to be green because they are not thinking of light traveling. They are thinking of the pattern on the wall but they're not thinking of the projector. When you think of a pattern on the wall, you're not thinking about the fact that it's being created by beams of light. When you're thinking about beams of light, you're not thinking *about patterns on the wall. Only when you get the two together do you say: "Oh yes, the shadow of the green light is red because the red light is shining right on that part of the wall and the green light isn't."*

It might surprise David to learn that even after I figured out the geometry of the colored shadows, I did not think of light traveling. I knew that the light went from the projector to the wall -- I associated what I saw on the wall with the source of the light. But I did not think in terms of light *traveling* from the projector to the wall. The light on the wall, somehow, was part of the light of the projector. I did not question how it got there.

White Light

After solving the colored shadows mystery, we turned to the questions raised by the mixing of colored light. There was less confusion but there were many questions. First, we had to learn to forget about pigments in order to understand the mixing of colored light, and that was difficult. Is this problem partly caused, I am now wondering, by our use of the word "color" in both instances? Would it help if we just talked about the differences in mixing *pigment* and in mixing *light*? But then we use pigment on the gels to make colored light. It is confusing!

Some of the teachers had mixed light by putting different colored gels over the different projector lenses; others wanted to see what would happen if they mixed, that is, superimposed, different colored gels over one projector lens. Cindy made an analogy between the dark, murky colors you get when mixing the primary colors of pigments and the somewhat similar effect you get by superimposing colored gels. She then tried to define what "adding" and "taking away" light means:

When we put the three gels on top of each other on one source of light, it got murkier on the wall. We were cutting out the light. But when we have the three lights separate, from three separate light sources, it was like "adding" light. When we added them separately, we ended up with white light. We're not just getting red, yellow and blue, we were actually getting more light.

I had also struggled with the concept of adding colors to make white light and after our third class I wrote in my journal:

I think in order to understand how colored light, when mixed, makes white light, you have to think of more and more light, *rather than colors being mixed. I just realized that that statement already presumes an acceptance of the fact that light -- when broken up, consists* *of separate colors, and when they are all mixed up again, you get "white light." You are mixing the separate components, getting back the whole. We've only had a few classes, but the "breaking up of light" into colors is being accepted by my brain.**

*Just now, as I was reading these notes, I realized that physicists must think very differently about light and color than I do. Even though I know that light can be broken up into different colors, I don't really think of color as being the property of light. I think that color is the property of the colored objects which I see. In the course this never became a problem for me because I wasn't far enough in my thinking to become confused. Jean, on the other hand, really struggled with this idea:

I don't understand how colors fit in with the whole spectrum that we call light energy. I don't even know how to ask about it. Colors are part of the visible light, and the color that light breaks into, that's energy? I don't understand that at all.

There were other interesting comments on white light:

Cindy: *I would never have thought that white light includes all other colors. Even as we saw the white light in the center of the shadow, I called it "absence of color."*

Sue: *I thought that we got white because all the colors cancelled each other out,...*

Polly: *I have great difficulty understanding why I get white light when I add colored light. Is it because each color is saturated with light and where they all meet, that is the most saturated, therefore it becomes white? Or is it that one color blocks out the other color and so on until you get white light?*

Because there were so many questions about the mixing of light, Ron got hold of a light mixing machine which had much purer color in its filters than we had in our gels. We worked slowly, all together, mixing first two colors, then all three colors. There really was white on the wall. This convinced almost everyone that the colored light, when mixed, produces white. Cindy was still skeptical: she wondered if the white was not just the color of the wall. If we were to project the colors onto a black wall, would we still get white?

After mixing light, we separated out the colors with the help of a prism. The brilliant spectral colors were greatly admired by everyone. We held up a second prism, which brought the colors together again. Ron then gave us a spectacular light show by hitting two chalk erasers together in front of a projector light and also spraying water into the beam of light. If you looked along the beam of light, little specks of color were swirling all around. If you looked across it, the light looked *white*. Ron explained that the different colors of light were hitting the little specks of chalk and the water droplets were bouncing off in all different directions -- "every little speck of chalk or drop of water is reflecting a different color." It was an amazing sight and further helped to convince the teachers that there really was color in light.

There were many questions about how a prism works. It *bends* the light, it separates the different wavelengths of the different colors, it organizes jumbled-up light, Ron said. Jean had a question about the effect of the prism:

If you bend light through a prism, that is organizing light in a specific way, is that red hotter or does it *have more energy than natural unorganized light? Could it do* things *beside be red in color that natural light can't do?*

After further experimentation and much discussion about prisms, spectral colors, and wavelengths, another problem came to the surface. The term *white light* bothered some of us. I was one of the people who had trouble with this, and I wrote in my journal:

I find white light very confusing. On the one hand there is the fact that light consists of a mixture of all the colors of the spectrum, and so white surfaces, white objects, contain all the colors. But to me that is color, not light. You can't ever see *white light. In fact, I have a hard time even thinking about light, except at night. Light bulbs make light, so do candles, fires, and all the things that light up the dark. During the day the sun gives us light. But although I sort of know that the sun is the source of light, when I look around in a room, or outdoors when you cannot see the sun, where is the light? I just realize that as I keep looking around to see the light, I am confusing light with air. Light is all around us like air. But by comparison with light, air is very real. I have an image of light seeping in, maybe between the air molecules, just sort of being all around and making things light. I would never think of* white *in that connection. I would like light to be* just light, natural *light. I know that it contains all the colors, but that doesn't make it white.*

David came to my rescue with regards to white light. He informed us that he was not going to call light *white* anymore -- "it's an incorrect word to use," he said. He settled for *ordinary light* and then continued:

Take a piece of white paper, like typing paper. If you shine colored light on it, it always takes on the hue of the colored light. If you shine "ordinary" light on it, it looks white. So white light is that thing which makes white things look white.

So much for white light! Terms can be changed easily enough. But there was still in my mind a difference between the ordinary light that's all around us, the ordinary light that makes white paper look white, the light that *just is*, and the light that you actually perceive as light, that you can look at -- the source of light, whether it is the sun, the moon, street lights or candles.

I did not think of these two kinds of light in the same way and I also had difficulty understanding that you only see reflected light. "Must light go from the light source to an object to our eyes?" I asked in my notes. "Is there no light just bouncing around in my room, like air? But you can *see* light if you look at the moon, or at a bulb in a lamp? So what *is* light?"

Marilyn, apparently, had a similar problem when she wrote in her notes: "I want to be able to relate white light to light bulbs, street lights, stadium lights, etc. How is light from these sources seen as white?"

Discoveries with Colored Gels

I was still puzzling about these two different kinds of light when I came up against another problem. In one class we were looking at different colors of construction paper and at multicolored book jackets through different colored gels. It was fascinating to see how the colors changed -- for example, green looked at through a red gel became black -- but I wasn't quite sure what was happening. What colors did the colored gels let through and what colors did they block?

When I thought about the projector lights shining through the red gel and coloring everything red on the other side, I was able to figure out that the red gel lets through only red light. But when I tried to transfer this knowledge to the situation where I held the gel in front of my eyes, I became confused. With an effort I could make myself think of light going from the projector bulb to an object, then to my eyes. Therefore, a red gel, placed anywhere on this path, whether in front of the projector or in front of my eyes, would allow only red light to go through. Yet in my journal I was trying to explain how a red gel *blocks* red light:

When I hold up a red gel to my eyes, everything "out there," beyond the gel looks red. Between my eye and the gel, however, everything is "white." Therefore, the red gel must be keeping the red light "out there" from coming to my eyes but allowing all the other colors of light to come through.

Then I made another discovery: I was trying to clarify what was happening with the gels, when I suddenly realized that much of the time I was thinking of light going *from* my eyes *to* an object on the other side of the colored gel. I wrote:

I never realized that I had been thinking about this in just the wrong way. How could that be? Did I think my eyes were little flashlights? Or is the idea of seeing being done by eyes so strong that I just assumed the source of light was in the eyes and that somehow my eyes were sending out secret little rays of light?

Some teachers apparently had similar problems. Sue commented:

I find myself looking at things and trying to realize that the image was being imprinted on my eyeball. When *you are "child like," you think that your eye initiates the seeing -- but actually it's the object, right?*

A friend told me that when he was little, he wondered if objects got tired being looked at, whereas Jean, when realizing that light comes *to* her eyes, expressed concern "about all the light my poor eyes have had to stop, absorb, over the years. I wonder whether that is related to poor vision as you grow older?"

Polly asked how *looking* through a gel related to light *shining through* a gel and then said:

I realize that I do have difficulty thinking of light coming to my eye when looking at an object. Somehow I feel my eye is doing all the work. Light is such a given, I forget that it plays an active role. I don't think of empty space as being filled with light; in a way I take light for granted.

It is interesting to note how often the teachers and I talked about having taken light or color for granted. This is how Marilyn reflects on the experiments with the colored gels:

I feel I have a better understanding of color and of what the colored gels do after the last class. It was interesting to hear Maja say that she had thought that the red gel let all the colors of the spectrum through except red -- that the red was "stopped." I had never thought that way. Actually, I had never really thought about colors of the spectrum entering the gel and being either absorbed or let through. It always seemed to me that things appeared red when looking through a red gel because of only *the red gel itself, and that there were no other factors involved. In other words, I never considered the light coming through a gel and going out -- what was being absorbed and what was not. Thinking in those terms was something new for me.*

From talking about light going through the colored gels -- transmission of light -- we came to ask questions about absorption and reflection. Light is reflected off shiny surfaces, David said, and absorbed by dark surfaces. He mentioned that he preferred to use the word "scattered" to describe light that is bounced back by non-shiny surfaces. To me scattered light sounded too random -- how could we see it if it goes off in all directions? "It *is* random," David said, "only a small fraction of the light that is hitting this table goes to my eyes, the rest bounces around in the room and eventually gets absorbed."

What happens to the light that gets absorbed? some teachers wanted to know. "When light is absorbed in a surface," David told us, "the light *as* light disappears and the surface gets warm. The temperature of the material rises because it has more heat energy.... When sunlight feels warm, you say there's heat coming from the sun as well as light. Actually, there's just more light. "If you're burned by the sun," Sue wanted to know, "are you burned by the light or by the heat?" David said:

Your body absorbs the light, your body stops the light. The energy of the light is transformed into the acceleration of the motion of the atoms of the material, which causes the temperature to rise. Heat is energy in another form. Absorption of light is the transformation of energy from one form to another.

David then told us of Benjamin Franklin's experiment in which he put different colored cloths out in the snow and observed that the white cloth melted the least amount of snow and black cloth the most. "He concluded from this that the black cloth was absorbing all the light and putting it into heat and that the white cloth was absorbing the least. But it's the fact that heat is produced that convinces you that light is being absorbed."

The heat is the evidence for the absorbed light! Suddenly I understood what had troubled me throughout the previous year's energy course. This new understanding represented an important stage in my learning and gave me great satisfaction: I had come to accept an invisible process through observable evidence.

After discussing how different colored materials absorb and reflect different wavelengths of light, some of us began to wonder about color itself. Why are things the color they are? We understood that a green object "absorbs" all the colors except green which it "reflects." But why is it reflecting green, rather than red, blue, or yellow? We got considerable resistance from the research staff to these questions but when we became insistent, David did tell us that color had to do with "the interrelationship between electromagnetic radiation and the electromagnetic properties of ordinary matter.... The fine structure of ordinary matter is such that it will be emitting radiation and absorbing radiation."

We now *knew* that color depended on the interaction between light and matter and if we could not understand this on a deeper level, at least we had a phrase to hold onto, and that helped.

After four classes on color, David was ready to leave this topic and to "retreat," as he put it, "to the much simpler subject of light and shadow."

Light Travels in Straight Lines: Shadows

For the next session, David and Ron set up a C-clamp in front of a light bulb. A string with a pencil attached to one end was stretched from the bulb, past the edge of the C-clamp, to a piece of paper on which the clamp's shadow would fall when the bulb was turned on. The string represented a single ray of light traveling in a straight line. A teacher volunteered to draw an outline of the C-clamp on the paper by pulling the string taut and holding the pencil on the paper so that while she was drawing the string was kept in contact with the C-clamp. The resulting drawing predicted the shadow which the clamp would cast when the bulb was turned on.

I wish I could transcribe the sounds of the laughter when the teachers realized the demonstration was supposed to illustrate that light traveled in straight lines and the scientist staff members realized none of the teachers had made that connection!

It would have been a good demonstration *if* we had understood that light *travels* and therefore can be stopped by objects placed in its path. Unfortunately we were not yet at this stage in our thinking. To us, light was not traveling and shadows had only a vague relation to light.

"I never thought about a shadow as absence of light," I said in class, even though I had already worked with this idea when I was solving the colored shadow puzzle (see p. 56). "How *did* you think about shadows," David wanted to know. I replied:

I don't think I ever really thought about them. It's just a dark area. I knew it was related to the object that cast the shadow and I knew that there had to be a source of light somewhere, but beyond that I never gave it much thought.

Hedy's experience was similar to mine:

I never thought much about shadows and I would never have thought of them as the absence of light, not in a million years.

Marilyn agreed:

I had always thought of a shadow as being something in and of itself, rather than the absence of something.... I would never have thought of a shadow as absence of light. That phrase seems to imply that there is complete darkness.

Polly was even more explicit:

I know that a shadow is caused because the light shines on something and somehow that causes a shadow but I always thought about shadows in a sort of positive way, as an imprint, or a photograph, or like the way you step in sand and leave a footprint.... I really thought of shadows as being "cast" not as something which was caused because light was stopped.

I added:

A shadow is a black image. It's not the absence of something. It is something, and it's dark and real.

In my everyday common sense world, an *absence* is a *lack* of something, a nothingness. An absence of light means no light. In a different context, for example, at night, I would have no trouble saying that it was dark because there was no light. But that is what Marilyn called "complete darkness." Shadows can be seen only when there is light around them, and often that light is very bright. Is that, perhaps, why it is so hard to think of shadows in terms of "absence of light?" Or is this related to the problem mentioned by David earlier (see p. 57) when he told us that while we were looking at the patterns on the wall created by the colored gels we were not thinking about the beams of light which produced these patterns? In Polly's analogy, a shadow was compared to an imprint, to something positive. An imprint is hard to reconcile with an absence.* If this is how most of us had been looking at shadows, it called for a radical change in our thinking.

After much discussion and further experimentation, this time with paint sprayed across a pair of scissors held in front of a piece of paper -- the paint representing the light rays and the outline of the scissors on the paper the *shadow* -- we came to accept the idea that light traveled in straight lines. But there was another problem: accepting that the light from the *bulb* traveled did not help us to understand that ordinary, natural light also travels. I mentioned earlier that I did not think about these two kinds of light in the same way. Some of the teachers were similarly troubled. Ordinary light, it seemed, wasn't *doing* anything. "You just think it's there, it's just there," said Marilyn. Polly wrote: "I took light for granted. It just was. I thought of it as coming from the sun, but also as just being part of our earth."

"You've got to come to terms with the notion that light is something that's in transit," David explained. "It isn't just sitting around stationary. Now that's going to come in conflict with your ideas about the light in this room because you don't see any light traveling around."

David knew that our confusion represented a major barrier and he devoted a considerable amount of time trying to help us to cross it.

*In our relaxed and informal class atmosphere this different way of thinking was responsible for a wonderful and often amusing sharing of scientific and naive ways of thinking about light and shade. In a regular science class, such a gap, if not dealt with, could cause serious problems for both teachers and learners.

Light and Vision

David introduced the next class with a brief talk on sense perception, comparing light and sound. Hearing and seeing, he said, are both examples of perception but you cannot think about them in the same way:

You can say "I hear a bell," and you can amplify this by saying "I hear the sound made by a bell." Saying you *hear the bell is a sort of shorthand for saying you hear the sound made by a bell. If you say you* see *a bell, you do not find it correct to say you see a sight made by the bell. That sounds queer. So hearing and seeing are not the same kind of thing.*

How could one amplify the statement, "I see a bell," Hawkins asked. We proposed, "I see the shape of the bell," or "I see the color of the bell." Polly concluded: "There's not a word that does for *see* what *sound* does for *hear*." David agreed but then brought up an example in which seeing does seem to parallel hearing:

There is one case in seeing which is like hearing the sound made by the bell and that is seeing a light. If I shine a light in your face, you don't say you see the flashlight. You say you see the light. In that case it seems all right to say the light is coming to your eyes. It's common sense language. In the same way, isn't it all right to say you hear the sound coming to your ears? Sound traveling is okay, isn't it? In the common sense world you say I hear a sound over there, but what you mean is: the source of the sound that comes to my ears is over there.

"Well, that's interesting," said Hedy, "I hear the sound of the bell, it's right here, but I see the bell and the bell is over there." David agreed. Vision is more complicated than hearing, he said, because in vision we sometimes think of light coming to our eyes as when a bright light is shining in our faces, but at other times, we think the object of vision is *out there*:

When we see each other sitting around the table, we're not seeing light, we're not thinking about light coming to our eyes. We're seeing objects out there and that's what we're directly aware of psychologically.... When we're talking about our own perception of physical objects in the world around us, we don't think of light traveling at all. I think we think of light then as what fills space. Let there be light. It's the opposite of darkness, and that's another conception. My hunch at this point, after listening to a lot of your comments, is that these are really two different ways of thinking and they very seldom meet. The light fills space and our eyes reach out to the objects in the world around us and give us knowledge about the world. That's a totally different thing from seeing lights and shadows or thinking about light traveling. The scientific theory is going to insist that there's only one kind of light. We're going to have to construct a picture in the scientific domain that will somehow correspond to these two very different ways of thinking in common sense. We have to reconcile these two ways of thinking *about light by insisting that there's really only one way of thinking about it.*

It was nice to have our common sense approach validated, but how were we going to change? Our view of ordinary light had served us well for all our lives but now it was in conflict with scientific thinking. At least David was sympathetic:

Think of the light in this room for a moment. The room is full of light. What happens when you try to describe this in terms of light rays? There are some coming in the window from the sun, indirectly scattered from the sky. There's some coming from these lamps. It's going out in all directions in the room. It's hitting all sorts of objects -- us and tables and chairs -- and bouncing off of them and being scattered in all directions. A tremendously complicated and random mix. And that's the physicist's version of the room being full of light. It seems terribly complicated compared to the common sense version where the room is just full of light.

David shared with us an old theory about light:

The favorite theory I have from antiquity is that light is something that chases away the darkness. Darkness is an obscuring medium, it keeps your eye from going out to things. And when you turn on the light or the sun comes up, all that dark mist is pushed out of the way. Isn't that a nice idea? There's a blackness that fills space and the light pushes it out of the way. The emptiness is transparent now and your eye can get through. It's a theory that is totally at right angles to anything present-day science would talk about but it's sort of a nice theory, it feels right somehow.

"Oh, that's so much better!" exclaimed Sue. We all felt comfortable with this ancient way of thinking. David then passed around an excerpt from Gerald Holton's *Thematic Origins of Scientific Thought* (Harvard University Press, 1973).* In the old emission theories of light, Holton writes, there is an intimate interaction through contact between the observer and the observed:

Plato held that as long as the eye is open, it emits an inner light. For the eye to perceive, however, there must be outside the eye a "related other light," that of the sun or some other source that allows rays to come from the objects...a coupling between the outer and inner world is clearly attempted.

The Greeks, the Arabs, and the medieval world, David said, thought it improper to separate questions about what's happening out there in the physical world from questions about what's happening inside the human mind.

*Holton discusses the old dual view of light with which we were struggling -- the ordinary, all-around light which allows our eyes to see, which he calls *lux*, and the other kind of light which is coming to our eyes from a source, which he calls *lumen*. Both derive from the same Latin root, *luminare*: *lumen* from the infinitive *luminare* "to light," and *lux* from the past tense *luxit*, meaning "it lighted."

They desired a unified picture of light and vision, all in one piece. Modern optics, he said, stops at the point where light forms an image on the retina. That is the end as far as physicists are concerned. They don't talk about what happens behind the retina:

Physicists will say that's a problem for psychologists and psychologists will discuss vision but not in terms that will match what the physicist is doing. There is no unity, no coherence between the two pictures of light and vision.

We didn't know that our thinking about light was so similar to that of the ancient Greeks. I liked their attempts to unify light and vision. If I were starting college now, I thought to myself, I might try to take a double major: physics and psychology.

From Pinholes to Light Rays

The topic of shadows introduced us to the concept of light traveling in straight lines: the two classes on pinholes consolidated our still-tenuous understanding.

To illustrate how pinholes work, David drilled a small hole into a door leading from our classroom to a large closet. A large board was put up in the classroom about six feet from the pinhole, and facing it. Several large alphabet letters cut out of colored construction paper were pinned to the board. In the closet, tacked on the wall opposite the door, was a sheet of white paper. We went into the closet, a few people at a time, and after our eyes got accustomed to the darkness, strong floodlights in the classroom were turned on, lighting up the letters and the area in front of the door. Inside the closet, we saw reversed and upside down images of the letters on the board. Then a teacher in the room happened to walk past the door and we saw her image, upside down, passing across our "screen." That was truly an amazing sight and somehow more convincing than the reversed alphabet letters. Everybody was really excited and the teachers took turns now going into the closet to see their colleagues walking, skipping, or jumping in front of the pinhole door.

Almost every teacher wrote enthusiastically about this experience. Here are some responses:

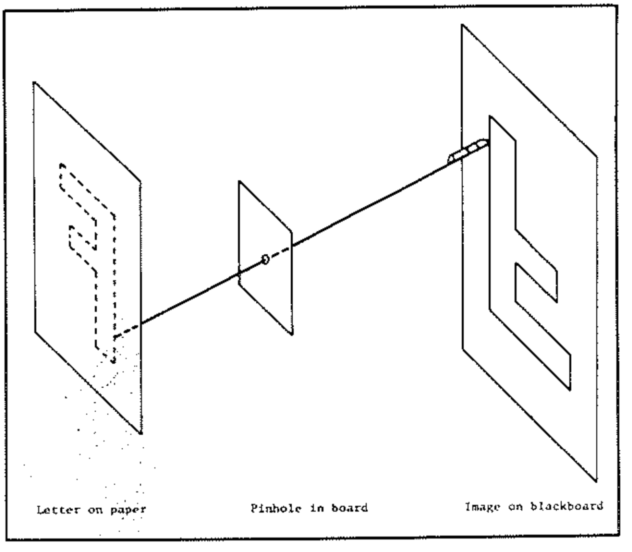
We got to be in a pinhole camera set-up. That was as wonderful as liquid nitrogen. I'm still amazed thinking of it. I can hardly believe such a miraculous thing as a pinhole image was on that paper but I have my experience that it really was! (Hedy)

I loved the pinholes.... At first I thought, how is that done, and was really blank, but the minute I thought of light traveling from the top right hand corner and in a straight line, it became clear in a flash. Understanding is always fun! (Polly)

I didn't know what we were going to do when we started talking about the pinhole camera.... I had no idea what we were going to see when we went into that room. I thought we were going to look out the hole.... When I realized what we were seeing, and how things were reversed, it blew my mind. (Sue)

Betty, who had done pinhole photography with her sixth graders, remarked that she was surprised to see color when looking at the screen in the closet. "I guess I just always pictured this *film* from a pinhole to be black and white," she wrote.

Some teachers understood what happened. Others needed to clarify for themselves why the image was reversed. To help us with the geometry, David and Ron devised a piece of apparatus which proved most successful (see diagram). A large construction paper "F" was



Drawing by John R. Taylor from a drawing by Ronald W. Colton.

pinned to a board which was mounted parallel to and about four feet from a blackboard. Between the "F" and the blackboard was mounted a small plate with a hole drilled in it. A dowel with a piece of chalk attached to one end was pushed through the hole. By following the outline of the "F" with the other end of the dowel and touching the blackboard with the chalk, an upside down, reversed "F" appeared on the blackboard. Even though all the teachers had seen the upside down image in the closet and knew it was formed by light coming through the pinhole, several teachers reported that they needed the experience with the dowel to fully understand what was happening:

It was very exciting to see the pinhole camera effect. And to figure it out! But it did not come to me until I traced the letter "F" on the board. Only then was I able to understand it. (Marilyn)

The work with the pinhole was the first time I'd ever thought through what does happen. I am excited about understanding this better. I was impressed with the apparatus for tracing the letters and the obvious connection with what happens with the light through a pinhole. I felt such a sense of satisfaction tracing those lines. (Jean)

When I figured out how light traveling through a pinhole produced a reversed image, I felt very satisfied. The idea of light going from a place, in a straight line, to another place was becoming quite clear. I even figured out how to make the image larger or smaller by moving the plate with the hole closer or further away from the letter "F." But then I again began to wonder about light: what is it really, I thought, that bounces off my head and carries an image to a piece of paper and from there to another person's retina? How do light rays carry images? I didn't understand that at all. I was thinking of a light ray as consisting of little particles of light that travel down from the sun in straight lines and I was trying to figure out how these rays *carry* images. When I asked David, he said that this was not the way to think about light rays. A light ray, he told me, was a geometrical construction with which you can predict; it was a useful model. "When we talk about light rays traveling in straight lines, we go beyond experience, we make a representation to explain the behavior of light. A ray is an invention of the mind." I was even more confused now. Light rays weren't real? I always thought when I saw a beam of light, extending from a flashlight, a projector, or coming down from the sun through clouds, it was made up of rays. Because my problems in understanding new ideas often were indicators of teachers' problems, David spent a good deal of time at the next class meeting discussing light rays:

I want to talk just briefly about one very central, very important, and in a sense very simple idea, but also in a sense a very hard one to accept, and that is this notion of a light source which emits light. You can think about the light emitted in terms of rays of light. The light ray is the basic abstraction used in all the thinking about light and vision. It is indeed a straight line because that just expresses the statement that when light is traveling through a uniform medium like air or empty space, it is traveling in straight lines. So you represent a tiny little bit of light traveling in a straight line by a straight line. In other words, it's just a symbol to represent the pathway.

How can you think of defining a light ray? Take a small source of light and then at some distance have a screen with a small hole in it. Then the light that goes from the source through that hole is like a pencil *of light. Don't pay any attention to the light that hits elsewhere, it can't go through the hole. Just the light that comes through the hole is that pencil of light which you can imagine as a very very small straight line, a ray. It's kind of an abstraction, an idealization. Any old geometrical straight line is an idealization. A piece of string has thickness but you pretend it's just a line.*

All of the everyday phenomena about shadows and pinhole images and lenses can be translated into the language of geometrical straight lines and you can make pictures of it and it gives you a kind of unified account. All of these phenomena you can work out for yourself once you get the idea that you're going to describe what's happening with this kind of geometrical representation. It's called geometrical optics and it does not answer the question, "But what is light, really?" It only talks about how light is transmitted. It only talks about the pathway and not about what *it is that follows this pathway. You see, that's enough. Why does light behave this way? Well, that's going to be a much deeper and much more difficult question.*

David had called light rays symbols, abstractions, idealizations; the straight line in a diagram is a geometrical representation, a model for a way of thinking about the behavior of light. This seemed strange to me. I can accept a line on a map as being a representation of a road or a dot on a chart as being a symbol for a star, but I can *see* roads and stars in the real world. How do I think about a line that represents something that is not only invisible but that does not even exist? Polly had written in her notes: "A big part of all my difficulty is that I can't *see* physically so much of what it is I am trying to understand."

David had said that the model of a straight line representing a light ray would help us to figure out a whole range of optical phenomena. That was not clear to me. Moreover, I would still only know how light *behaves* when I wanted to know what light really *is*. How will an understanding of geometrical optics lead me to an understanding of the nature of light? David had mentioned the book *The Philosophy of Science* by Stephen Toulmin (Harper, 1960) and in it I found a description of the role of models in physics that I found very useful. Toulmin discusses the nature of discoveries in the physical sciences, using geometrical optics as his example. He writes:

The discovery that light travels in straight lines... was a double one: it comprised the development of a technique for representing optical phenomena which was found to fit a wide range of facts, and the adoption along with this technique of a new model, a new way of regarding these phenomena, and of understanding why they are as they are.... The very notions in terms of *which we state the discovery, and thereafter talk about the phenomena, draw their life largely from the techniques we employ. The notion of a light ray, for instance, has its roots as deeply in the diagrams which we use to represent optical phenomena as in the phenomena themselves: one might describe it as our device for reading the straight lines of our optical diagrams into the phenomena. We do not* find *light atomized into individual rays: we* represent *it as consisting of such rays.* (p. 29)

This gave me a beginning understanding of the relation of models to invisible phenomena in the world though it required a kind of thinking that I am almost completely unfamiliar with.

Toulmin also helped me with my question about how I would get closer to an understanding of the nature of light by starting with geometrical optics:

One might speak of models in physics as more or less "deployed." So long as we restrict ourselves to geometrical optics, the model of light as a substance traveling is deployed only to a small extent; but as we move into physical optics, exploring first the connexions [sic] between optical and electro-magnetic phenomena, and later those between radiation and atomic structure, the model is continually further deployed. (p. 37)

A good model, Toulmin says, is one that suggests further questions, that takes us beyond the phenomena from which we began, and tempts us to formulate hypotheses which turn out to be experimentally fertile.

David put it this way in class:

You invent a hypothesis and then it becomes a powerful descriptive tool. Scientific theories are inventions that we credit and believe in and support because they unify our experience but not because they are somehow the *absolute truth. They are models, they are representations. They must have something right in them or they wouldn't have the power that they have. The fact that they predict and unify all kinds of otherwise unrelated experiences is terribly impressive. And yet fifty years later they say, "well that was a very good theory but now we can reconstruct it..."*

We were not only learning about light and color in this course. We were learning about scientific models and scientific theories, about new ways of thinking which were necessary both for specific understandings and for the much broader understanding of the development and the history of science.

I was so pleased about my better understanding of models and theories after listening to David and reading Toulmin that I decided to venture further. I took home an old copy of the *Scientific American* (September 1968) and tried to read Victor Weisskopf's article entitled "How Light Interacts with Matter." I understood just enough of it to know that there *is* a reason for the color of objects. That was exciting. In the evening I wrote in my notes: "A lot of things are falling into place...."

Postscript: Images

In my earliest thinking about light, in the energy class, I wondered whether light flowed, like heat. Later, in the seminar on light and color, I found myself confusing white light with air. Both are *all around us* and are taken for granted in everyday life. When I realized that this neutral kind of light was quite different from air, I had trouble relating it to the light that comes from a lamp or from some other source. I found out that the Ancients thought about light in two different ways, but that in modern physics there is only one kind of light. Then I learned that light travels, and that *all* light is constantly in motion, "in transit" as David put it. While this idea was becoming acceptable to me, I tried to understand that a light ray is a model for the behavior of light and I figured out the geometry of the reversed pinhole image. I grappled with the understanding of absorption, reflection, and scattering of light and I even obtained a rudimentary understanding of what accounts for the color of different objects. But I still didn't understand *how* light actually produces images.

All through the seminar, and all through the writing of this essay, I was thinking about images as if they were pictures. In some vague way, I thought of all the little particles of light scattered off an object as somehow *carrying* in a straight line a tiny part of the picture of the object to where the image was formed. In my mind, the pieces of this picture were traveling through the air on light rays! This formulation strikes me as so ludicrous now that I am almost too embarrassed to report it. I knew this wasn't really how things worked but I didn't know how to think about it. Did the language -- words like *traveling* and *carrying* -- throw me off? Or were all my points of reference to pictures?

I became so frustrated by my confusion and so impatient to understand this that I phoned a physicist friend of mine, long distance. He explained to me that images are formed by light -- more or less light and light of different colors -- and that the images we *see* are representations of the *pattern* of light and color produced by the objects from which light is scattered. Obviously this must have been discussed many times in our classes but it wasn't till I had almost finished writing about the course that I became aware of my confusion about images. My old question -- how does the image get from the object to the paper (or the eye) was finally answered.

Implications of the Research

"When have you planted enough clues so people can ask questions?" David once asked at one of our meetings. There is a stage in learning when you don't have any questions. The words or concepts have no meaning yet, the confusions haven't been uncovered, you don't know in what direction the learning is going to take you, and therefore you have nothing to ask. What is the teacher's role at this stage? For me, this is a *taking in* period -- hearing about the topic, listening to discussions, doing some experiments, reading, and gradually letting the new ideas sink into my consciousness. Then one day I may surprise myself with a statement that shows that I understand more than I realized. At this stage I appreciate having a teacher pose questions which plant clues, which arouse my curiosity, focus my learning, make me aware of things I hadn't noticed before.

Questions

There is a subtle dividing line between leaving students alone to make their own discoveries and telling them just enough so that they can proceed on their own. If you tell too much, you deprive them of the pleasure of making their own discoveries. Answering *your own* questions leads to the most meaningful and lasting learning. Marilyn wrote in her evaluation:

At certain times during the course I felt angry about not being given more information. However, now I am grateful that I wasn't deluged with facts. If I'd been given too many explanations, I'd never have had the positive experience of discovering concepts on my own.

Hedy described what learning situation she liked best:

None of the experiments which were set up for us to observe had the impact for me of that initial "just playing" with the colored gels, when Ron came by and raised questions, gave us "clues" but did not answer our questions. That seems like the best model. When experiments were set up to teach us something, it was very different from playing around and stumbling and questioning and coming out with a "burning desire" to know.

I know what Hedy means when she talks about having "a burning desire" to know something. The closer I come to understanding a new idea, the more urgently I seem to want an immediate answer to a question. I often wonder about this sense of urgency. Is it that I am afraid I will forget what I know? Or is it that I am at the end of a long struggle to understand and when the goal is finally in sight, I want to get there fast to resolve the tension? Polly once described her experience in the course as being both frustrating and exciting. Doors were opened, she said, and suddenly you realized how little you knew and how much you would like to know. "The process of learning is fun but I want it to be leading somewhere." Once you find out that there is a way to understand something that you have always ignored or considered inaccessible, you become impatient to know. We were at times quite insistent in class about wanting answers which we thought would be of help to us. When we got an explanation, we often did not understand it. Our knowledge and experience were too limited to process the new information. One day David tried to explain his reluctance in giving us quick answers:

We honestly don't know how to answer some of your questions. They are very insistent and very hard to deal with because the typical way of answering them makes use of ideas that haven't been developed yet.... An awful lot of what is called science is the outcome of people wrestling with problems over a long period of time and gradually developing pictures that are not the ordinary ones. The attempt to translate all the things that happened in that development into a few well-chosen sentences just doesn't make sense.

There were certain kinds of questions which we tended to ask which troubled David: "What is electromagnetic radiation? how do light waves work? what exactly are photons?" and so on. "If you are thinking about a subject which you understand very well," he said, "do you ask for explanations of the kind, 'But what is light really?' If you're thinking about children's minds, for example, do you ask, 'What are children's minds, really?'" He continued:

I think you recognize that you're constructing and choosing models, images, useful metaphors, and devices for how you think about these things. And you're very conscious of the fact that day after tomorrow you may revise your picture.... With anything you really understand and come to terms with you recognize that you always have more to learn and that therefore you don't have a final explanation. The more you know the more you're like that. Whereas when you have a passing interest in something that you feel you don't understand you want to get an answer that eliminates further questions. We've all been frustrated by so-called scientific explanations and we'd like to be relieved of that frustration. I don't think you can quite be relieved of it, you've got to get used to having it. Then you slowly build up a picture that is more and more satisfying but never with the feeling that you've said the last word or that anybody can tell you what the last word is.

Because several students had expressed frustration about having answers "withheld," David posed the following question in his request for a final evaluation paper:

"We have left some topics hanging, inadequately described or explained, such as light waves, resonance, lenses. How do you now react to our diffidence about plunging forward with explanations?" Interestingly enough, by the time the course was finished and the teachers had had some time to reflect upon their experiences and their learnings, no one seemed to mind having been "left hanging." Here are excerpts from the teachers' answers to this question:

These dangling questions don't seem like difficulties now, just more to know about. That's very positive for me because I don't have to understand them right away. I'm under no pressure to unravel them. (Hedy)

I expect to have lots of loose ends. This is what this is all about and also the state of my understanding of concepts. I'm prepared for that to be the way of things, not only with this seminar but with how I operate in the world. (Jean)

Not truly understanding light waves is another thing to which I have resigned myself. I feel overall that I have learned so much, therefore it is acceptable to leave a few things hanging. (Polly)

I had a hard time learning to accept the open-endedness of science. "Does everything have to lead to other, more complicated questions?" I once asked. It took me a long time to become comfortable with this lack of closure. The more I know about a subject, the better I can accept open-endedness, probably because my existing knowledge gives me the security to be left hanging. When I delve into new subject matter, my tolerance for open-endedness decreases noticeably. I feel insecure with my lack of knowledge and, as David said, I seek answers that eliminate further questions.

Students must be helped to accept open-endedness, ambiguity and temporary confusion as a normal part of the learning process. They will then be better prepared to tolerate the discomfort that so often accompanies incomplete understanding and knowledge.

Feelings

Learning is never just a cognitive activity. The teachers experienced many highs and lows during the course -- satisfaction, pleasure and excitement when understandings were reached; confusion, frustration, even despair when ideas remained inaccessible. Such intense feelings are rarely expressed in traditional teaching situations, yet they played an important role in the learning that took place.

Difficult material often produced a feeling of overload. When the saturation point was reached, no more learning could take place. "I am tired and it takes too much of an effort to understand this;" "I didn't even try to understand that -- maybe another time;" "I don't want to deal with this right now." These were typical comments. I sometimes felt in myself strong resistance to getting back into thinking about light after being away from it for a week. "Do I have to struggle with this again?" I wondered. Yet as soon as I was in class, this resistance quickly vanished. One week Jean wrote:

At one time I needed not *to come to class. This happened when I was feeling particularly confused with putting together what I was observing and hearing. It was such a dilemma for me. I felt I needed time to absorb and not to work with equipment that week, yet I knew I would be missing so much.*

The pace of our classes often seemed slow to the research staff, yet teachers were asking for more time: They wanted more time to work with materials, to explore things by themselves and at their own pace, to let ideas sink in. "I like to think about the class for a day or two and then I find I wake up with questions and ideas coming together," said Sue. And Jean wrote:

I need to hear things more than once, or to hear them in another context, or to have some exploration time and to hear them again, or to have some discussion and then hear again with different ears.

The need for repetition was expressed frequently:

I wanted to play more with the pinholes, to come back on other days and do it some more and some more, for the part of me that needs repeated experiences of the same phenomena to really believe it. (Hedy)

Mary was frustrated when new information was given "just when you got something, then you get confused all over again." Hedy mentioned another reason for slowing down: "I didn't have any interest in the other questions," she said, "I was too content with my new glimpses." I remember often feeling the same way when I reached some new understanding. I wanted time to savor my success, to enjoy having reached a plateau. All too often, teachers get carried away by their own enthusiasm when students are learning and do not allow them sufficient time for this kind of celebration.

Enormously important was the climate we created in which teachers could feel free to admit their confusions at the most elementary levels of understanding. Yet even in our safe and supportive atmosphere, trust was not always easy. Sue wrote:

I wonder if you realize the level of trust that is needed as we write these stream of consciousness papers. It's one thing, and rather easy, to say: I don't know, or I don't understand. It's quite another thing to *expose yourself so completely, to explore all the ways you don't understand, not monitoring any of your questions.*

Why did the teachers come back, week after week, to struggle with science? I believe it was the gradual access they were gaining to a new and exciting world. When you begin to believe in your own capacity to learn and understand, you are no longer completely dependent on your teacher. The anxiety and frustration that often go along with such dependency are replaced by feelings of confidence, power, and joy.

Teaching, Learning, and Thinking

The seminar on Light and Color was a very special course. There were three instructors for ten students. There was no time pressure to cover prescribed topics. There were no concerns about grades. Classes were informal, discussions free and open, the climate supportive. The participating teachers had limited experience with science and were grappling with a difficult subject, but David had convinced them early in the course that their troubles and confusions were of great interest to him and his associates. The teachers were respected, their thinking was valued, and they soon developed a trust in the staff.

In a typical science class, whether for adults or younger students, the instructors work with many more constraints. In a course in which a definite amount of material has to be covered, it is difficult to *uncover* students' problems. In the energy course, which I audited as part of this research, the instructors were often concerned about falling behind in their schedule because many students were having difficulties with the course material. A month or so after the beginning of this class, a student commented to me how little material had been covered. He appreciated the slow pace and the instructors' patience with student problems: "They could have covered a lot more," he said, "but then we wouldn't have understood anything."

Science teachers may become frustrated when their students fail to learn and students tend to become frustrated when they cannot understand what is being taught. In traditional science teaching, the burden is on the student to understand the lesson. In a class which focuses on barriers to the understanding of science, the burden is on the teacher to present material in such a way that students will be able to learn. If teachers could become more interested in their students' thinking -- no matter how elementary -- their frustration might change to fascination.

David was always interested in our confusions because they taught him something about a more naive way of thinking about science which he knew was widespread but poorly understood. As students, we valued his attitude enormously. "I truly do appreciate being treated as a person whose ideas and thoughts receive credibility and respect," Jean wrote in her final evaluation. "That, as much as anything, is what is important to me about this approach and what has the greatest impact upon my teaching and dealing with younger humans."

The courses had a powerful impact on almost all the participants. In her evaluation of the Light and Color course, Sue wrote:

I never questioned. It says in my 4th grade science book that light travels in a straight line. I accepted that as a child accepts the faith of someone they admire and respect. I am changing. My mind is opening up more since we started the class. Now I think: What other ways could light travel? How do things go from one place to another?

Hedy, who attended both semesters, shows how her feelings about learning science changed over the course of the year. During the first semester, she wrote:

I don't have a real belief that I could understand science. I don't reach out because I'm afraid I won't understand and then I'll be hurt. I have never felt friendly towards those subjects.

At the end of the second semester, she said:

I feel as though I have never in my whole life had a satisfactory learning experience in science until now .... Many topics of a scientific nature now seem more accessible to me.... I feel better about myself, more powerful. It shakes the old negative ideas of myself to enter into new understanding.... I am immeasurably broadened by both semesters, doing something I haven't done before.

I feel wistful. I've had a little taste and now I'd like more. I love having been led into this. My understanding is tiny but it seems like more than I've had in my whole life.... I would like to go on indefinitely, meeting weekly and exploring new ground.

Many teachers, like Hedy, mentioned that they wanted to continue their studies, but most of them did not want to take regular college classes. "Where do you find a similar learning situation?" they wanted to know. We could not tell them. The research, however, had confirmed that given the right set-up and approach teachers who had previously avoided or feared science soon became excited learners with a strong desire to continue their scientific explorations. It is possible, in a relatively short time, to bring about a significant change in students' attitudes toward learning science and toward the role that science can play in their lives.

4

*Knowing Where You Are: A First Essay Towards Crossing Critical Barriers**

*This chapter was written by Philip Morrison. Philip Morrison is a world-renowned physicist at MIT. He has been involved in primary school education for more than 15 years and played a role in the development of the Elementary Science Study and African Primary Science programs. He has written widely in science and, most recently, was co-author, with Phyllis Morrison, of *Powers of 10*.

Before the Kalahari Desert was divided by a high wire fence and patrolled by armed jeeps, the San people dwelt there, as some still do, following their unending seasonal rounds under the sun and the stars. All that they materially possessed they carried with them. That inventory was small indeed, but their minds were full. Wanderers, though never aimless, they depended upon and enjoyed an intimate knowledge of every feature of their wide and lonely range. Every knoll, every rock, every sparse patch of growing things, bore its proper name and its own bundle of memories. As the band made its way, the very landscape steadily evoked a shifting and lively conversation. They could explain their untiring interest in the world, too: "We always like to know exactly where we are," they said.

Enlightenment philosophers like John Locke held the view that the human mind is given as a clean slate upon which a suitable scheme of education may write whatever it wills. That view is wrong, wrong for desert gatherers, wrong for schoolchildren, wrong in particular for student teachers and for their more experienced colleagues. Certainly by the time the individual has mastered any natural language, the mind's slate is well-formed and full. Plenty more can be learned, of course, but the learning process is not at all like writing on a blank piece of paper. Rather it involves annotating, amplifying, enriching with example, supplying new procedures for entry and confirmation, adding links and illuminating comparisons between portions earlier seen as distinct. Sometimes, to be sure, the blue pencil is needed, and cutting and editing improve what is there.

The accounts in this monograph of critical barriers to the learning of science by beginners demonstrate and document this general view. No one comes to a college class without a large body of quite workable theory. For not only wanderers but most of us in modern society also seek to know exactly where we are. Not knowing is uncomfortable, often dangerous. Language, even old saws, custom, direct or half-remembered advice, everyday experience with a multitude of images, locutions, daily tasks, and interaction with a variety of people, intimates and strangers, provide everyone with a conceptual and generally factual framework for coping with the world. We call it common sense.

Quite a good deal of that treasury is built-in to the circuitry of the brain, and more is selected out of experience. Consider, for example, the perceptual hypotheses that underlie most of the visual judgments we all make to cross a busy street or to catch a ball or even to reach out for a pencil. Robot designers have a way to go before they can emulate the structure of kinematic and topographical theory held by an active five-year-old, most of it of course not at all open to easy expression, but constantly seen in action and often recoverable by study.

Common Sense and its Correlations

The critical barriers that David Hawkins describes are a working part of the well-furnished mind. Some of them at least are remarkably unvarying. Hawkins has explicitly shown that some critical barriers met in his Boulder classes express the very views of Aristotle and the schoolmen, the cause of difficulties for beginners in science that are often more grave -- and certainly more interesting -- than the merely "pedagogenic disorders" also reported in plenty.

There is little mystery about this persistent structure. The social anthropologists, the linguists, and the psychologists find it as grist to their many mills. For one who does not engage in producing or managing the technical bases of life in either epoch, daily tasks differ little between old Athens and today's suburb. Routine physical chores, reading and writing, the complex web of relationships with family, friends, even pupils in the classroom: all those depend upon many skills of hand and eye and on habits of mind and speech, skills that are predictive, effective, and widely shared. But they do not differ so much over the millenia. There are more images today; the TV set is prolific as the fresco painter never was, but to one who has only to regard an image, and takes no more part in its electronic distribution than to buy the set and push the *on* button, only the nature of the surfeit makes much difference. The screen is otherwise like a decorated page.

Both the necessary perceptions and the deft handling of the essential artifacts of daily life do not much differ whether the vessel of cool water is formed of clay or of the latest petrochemical plastic. Using the telephone and driving a car are unique to our time, but the swift hints of the conversation itself and the quick reflexes so neatly trained do not add much new to the demands of life as it always was for those so placed in society that they did not daily make some specialized tool-using intervention into the physical world. Human relationships carried through stance, word, gesture, and deed, differ even less. So arises common sense.

What sort of content does common sense hold? The air is invisible but always ambient; when one needs it freshened, the window is opened. Yet a glass ready on the shelf is, and always was, regarded as a matter of course to be empty, never as filled with air. A dropped ball comes to rest somewhere nearby, and cannot bounce along forever. By day, light is present to make the space useable; when daylight is lacking, one turns on the lamp, a notably shining supply of needed light. Workable, often complicated and subtle judgments about homely detail tell us whether things will fit, can be moved, will hold water or keep out the cold. Little needs to be measured or drawn; at most a length of cloth is suited to the width of a table. The stick of butter and the cup of sugar are the most used metrics; the gallon of gas and the passing time are digits well displayed. Images dwell on the page or the screen or on the mirror surface; what we most often see in a mirror is placed directly before us. Trees grow taller over the years, and by their nature stay somehow in proportion. One senses too hot a room, or too cold, and makes appropriate changes in the circumstances. Perhaps a thermometer aids that judgment. Road signs and speedometers work roughly for anyone able to read. Other instruments are rare: neither a shadow marker nor a measure of wheat are as common as perhaps they were in everyday Athens.

The conclusion can be summed up in the remark that a complicated set of judgments based on experience and reason has always gotten people through many days. The matters at hand are ordinary enough so that a subtle correlation of size, nature, motion, energy flow, or whatever it may be, is almost always present. The appropriate presence is assured by built-in design, mainly due to others. Perhaps it is based instead on the distillation of long experience, enabling common sense actions to work with good probability. They seldom need explicit calculation, nor is there any desire to pose sharp logical tests of the comfortable and usually adequate presuppositions for action.

These everyday encounters -- and the theories, explicit or unvoiced, that alike arise from and motivate them -- are rarely analytic. More often they are syntheses, built of several mutually weighed discriminations. They are diverse, not often seen as unified by logical inference. They come out of practice, and nowhere is much effort made to fit them into a simpler whole. In general that issue does not even arise, for what is involved are rough conclusions about a wealth of distinct details, rarely step-by-step paths to long-pursued ends. They have more to do with the comprehension of whole events than steps of understanding. The cup drops on the tiles and breaks at once, or else it remains intact after a fall to the carpeted floor.

There is no need for, or gain from, a sequential analysis during the microseconds when the crack is spreading around the glass. The light goes on, or the engine starts, once the right switch is activated. That chain of familiar but unseen events is not analyzed, but taken as the effect of one initial cause, one willed action of the hand. Often small accompaniments are needed, found by precept or experience, that improve the result. A key is to be held a certain way, or a lid pressed a little to one side. That is often the closest the actor comes to reflective analysis.

But the complicated everyday world works, mostly, and therefore the point of view we take of it is naturally satisfactory. Neither generalization nor objectivity nor precision are very important to the common-sense frame of mind. Given some willingness from time to time to improvise and modify, usually by trial and error once faced with a new artifact or condition, some novelty can be incorporated. That new learning is soon naturalized by repetition within the old domain of common sense. Anomalies, all the way from transient minor puzzles to ghosts and miracles, are rare; even when they occur, they are mere happenstance in a complicated environment, to be weighed against a lifelong chronicle of the successfully ordinary. Let a real mirage or a deft magician bring in something genuinely unexpected; even then we are prepared by experience to discount the unusual event. After all, it is unimportant in the long run.

Such is the theory of knowledge we all share under the rubric of common sense. It is the expected general correlation among varied experiences that validates our views: the light is almost always there in the room without delay once we snap the switch or kindle the candle. Whence and how light moves is not even asked and would indeed be hard to answer through our commonplace perceptions.

Beyond the Range of Common Sense

In context, there is plainly nothing wrong with the view of the natural world that seems embodied in what I have called common sense. It is a broadly workable first approximation to a limited but extremely important set of phenomena; what you could call the everyday. It is, moreover, strongly reinforced conceptually by the common language, for what are pretty surely deeply evolutionary reasons.*

*I am referring not at all to biological but solely to linguistic change.

But an understanding of even the beginnings of natural science requires one to transcend this range of experience. Is it the sky? Distance and gravity and inertia rule its perpetual motions, all outside of usual sensory reach. Is it the perception of light? Time is, so to speak, absent for vision, yet light propagates, as the physicist knows but common sense never needs to admit. Is it air? Invisible, it nevertheless has weight and major chemical effect -- life-giving, in fact. Breathing is hardly comprehensible on common sense alone, a strange exercise of in and out, changing nothing, yet somehow intuitive and essential. Is it weight? Fundamental for anyone who changes the form of matter, every recipe touches upon it. The craftsperson who works at larger scale or with any but the most customary and routine materials depends on it pretty strongly. Its remarkably simple linear behavior violates all the rules of more complex and common-sense quantities; it never saturates, it never hits diminishing returns, it never is deceived by screens or walls. (The buoyancy of air does affect it at a delicate level.) Heat? These classroom studies make plain how complex a perception that becomes, once experience tries to deal with anything beyond the tried and true arrangements of clothing and space heating. The camper in winter, the firemaker, the careful cook, already need more of a guide to heat transfer than the feel on the hand.

Bumping Against the Barriers

It seems useful to cite examples of collisions with critical barriers. In a way here is the very heart of the matter, for the recognition of such collisions is usually the chief basis of their identification. If what I see is borne out, these collisions do not represent so much the inadequacy of logical categories and operations -- something akin to the stages made famous by Piaget -- but rather the systematic, if less than conscious, application of workable and even tested common-sense theories within domains where their utility fails. At least one major cause of such misapplication is the lack of experience in the novel range of phenomena. A second factor appears to be the powerful effect of ordinary language, whose statistically-reliable connotations often mislead a user extrapolating to a new domain.

Consider a few cases of this curious yet entirely explicable narrowness of everyday theory. One student felt that the shadow could not be described as mere absence of light. It made so strong a mark, she felt, that it must be an active presence. Yet striking graphic results are produced every day by simple tricks that merely keep out the marking substance in order to produce a figure-ground contrast. The cave painters made pigment patterns around hands placed on the wall! Painting with stencils and resist dying and a dozen such tricks would suggest that contrast is a means to a recognizable mark made by nothing at all. Experience with contrast can extend the common-sense generality that strong effects need a physical carrier: the perceptual difference is physical, but the figure itself may be a sign of absence.

The travel of light is not rigorously demonstrable without explicit instrumentation, no doubt. But it is made pretty suggestive by the use of a small hand mirror to direct sunbeam and lamplight into dark spaces, or the view of darkness outwards to the lighter space. A cloud of chalk dust makes the idea of a ray more real; talk of direction of view might strengthen the insights of those who felt, quite as do the expert mineralogical and chemical guides, that the milky appearance we call white and the clear passage through substances we call water-white are two distinct colors. The *white* light of the simple text accounts of color vision does not refer to milk-white at all; it is a jargon name for light that resembles the unbiased color balance of sunlight, called white in a second sense, that it is not the deliberately filtered green or red light of color experiments. Here it is language that enters to make marginal experience ambiguous.

As for scale, it enters everywhere, below simple perception and above it, too. Moving heavy weights or painting barns or building kites or using a crystal radio or seeking to explain breathing and fire all bring home one way or another the domain of scale and its dimensional attributes, indispensable to science down to the atom and out to the stars, and really prior to both.

Widening the World of Experience

In this brief reaction to a wonderful diagnosis no claim can be made for an easy cure, but I see a direction to move. To be sure, it is no novelty. There can hardly be a better way to lift the common-sense barriers than to extend experience beyond the everyday. Almost any direction that extension takes is full of gain. To paint the floor, to build a kite, to study arithmetic and geometry, not with symbols alone but with rods and string and paper, to cook for a dozen at table, safely to move a piano or a sand grain, to make a blueprint image in the sun...in short, to open yourself to new experiences, not carefully worked out by others beforehand, allowing time and concerted attention enough to permit explorative trial and error. The world to search is the world of almost every craft and sport pursued for utilitarian reasons or for simple enjoyment; but it must be entered across some easy threshold of action and not by images and symbols alone.

Did someone once refer to messing about? It is messing about -- not turned to a single clear purpose, but focused on some real physical system at hand, as Rat kept his boat at the river's edge -- that offers the best crossing of barriers, to my mind. For messing about with boats, for example, can and will come to include knots and waves and reflections in still water and cold hands and the spawning of frogs, and even adzes and map projections, if given time to grow.

If the prescription sounds as much like art and craft and sport as it does science, all the better. There is need as well for the more academic "language arts" (if the jargon persists). What is called for there is practice in the communication of concept and action and values by a wider variety of means than words alone. Diagrams, maps, sketches, teaching of some manual skill hand on hand: those belong to human communication as much as do reading and writing (not to forget written recipes and protocols), and are perhaps even more relevant to the beginnings of science.

Finally, we scientists and especially authors of textbooks need to lend a hand in this process. We too often use time-honored words that connote metaphors not to be grasped, like *heat flow*. We freely employ diagrams and sketches without serious effort to develop a common visual grammar. We grow impatient with approximate description and partial understanding even on the way to a working grasp. In arithmetic instruction it is notorious that the precision of algorithms good enough for accounting or number theory, but employed instead to overspecify rougher statements about the world, has induced a widespread and chronic pedagogenic disease of right answers. Neat formulas came for the candid teachers in Hawkins' seminar to mean *area* itself, where area should suggest something more like *ideal paint*, and the formulas mere short cuts in simple cases.

All that can be changed. Change requires an understanding of what this collection of close looks at real learning in the classroom tells. There is a good wind astir, perhaps only Force 3 right now: "leaves and thin branches move constantly, a flag flutters." But the wind will strengthen. It must bring more than symbol, more even than images on the screen, however nimble and interactive they are made. We need to place hand, eye, tongue, and mind all together to work upon the real world. We need to invoke the shimmering variety of experiences that border upon and can extend the complex but well-worn patch of daily life for which the student is so well prepared by common sense.

People indeed like to know where they are; the trick may be to lead them to many agreeable places, until they recognize that they too can begin to know and to feel at home in almost any domain where other human beings have dwelt in pleasure.

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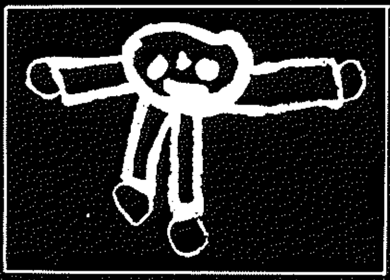

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